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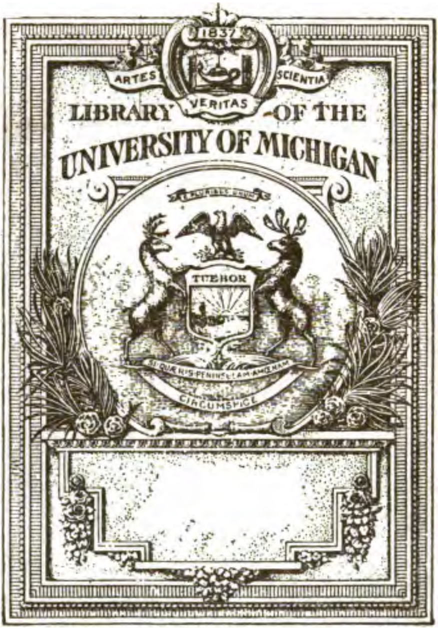
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NEW ELEMENTS OF *Conick Sections*:

Together with a

METHOD FOR THEIR *Deſcription* on a *Plane*.

Tranſlated from the *French Treatiſe* of
Philippe 1640 — 1718
Monſ. DE LA HIRE.

By BRIAN ROBINSON.

The Third Edition.

LONDON:
Printed by *T. W.* for D. MIDWINTER
in St. *Paul's Church-Yard*.

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4-27-1923

To the Reverend

Mr. *John Harris*, A. M.

AND

FELLOW

OF THE

ROYAL SOCIETY.

SIR,

SINCE *Mathematicks* hath been introduced into *Phyſical Speculations*, and Men have begun to ſee through the empty Jargon of the Schools, to diſtinguiſh betwixt *Chimeras* and *ſolid Knowledge*, Learning has received vaſt Improvements: And

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13 Dec. 13 [illegible]

And I may venture to ſay, that you, by your late excellent *Work*, with which you have obliged the World, have done more Service to *true Philoſophy*, and *uſeful Learning*, than any one ſingle Perſon beſides; for thoſe many and great Advances which have been made within this laſt Century, in particular Parts of Learning, you have with great Improvements and good Advantage brought into one View, and under the modeſt Title of *Lexicon Technicum*, have given us a compleat Body of Science.

Nor is your Goodneſs leſs conſpicuous than your other Qualifications; your generous *Converſation* and diſintereſted ſincere *Friendſhip*, makes you eſteemed and beloved by all thoſe who have had the Happineſs of your more intimate Acquaintance; and I am ſure, I may with better Reaſon ſay of you, what that General *Alexander* did of his Maſter *Ariſtotle: That my Being indeed is a Debt due to my Parents; but my Well-being is entirely owing to you.*

Be pleaſed, Sir, to accept with your wonted Goodneſs, this ſmall Preſent which I here make you, and look upon it as a Mark of that juſt Eſteem and Regard I have for my beſt Friend. I own it is yours by Right; for it was you that engaged me in it, and have ſince reviſed the Sheets as they came from the Preſs.

I need not ſay any thing of the *Author* I have here tranſlated; for your putting me upon it ſufficiently ſhows the Value of the Original, and your correcting the Sheets, in my Abſence, will, I hope, in ſome Meaſure recommend the *Performance.*

DEDICATION.

May you live long, and happily, for the Service of the *Church* and your *Country*, the Encouragement of good *Letters*, and the Ornament of the *Mathematicks*; which is wiſhed by none more earneſtly than by,

SIR,

Your moſt Obedient,

and Humble Servant,

B. ROBINSON.

The AUTHOR's

PREFACE.

SOME Years ago I publiſhed a Treatiſe of Conick Sections in a new Method, and demonſtrated their Principal Properties from the Cone: But ſuch as had not been ſufficiently enured to Demonſtrations about the Interſections of Planes *and* Solids, *found it difficult to underſtand them, though they were very ſimple and plain when once comprehended. This put me upon ſeeking out another Method, whereby, without making uſe of the Cone, and by only drawing Curve Lines upon a* Plane, *I might demonſtrate the ſame Properties of the Conick Sections; and after having tried that Method, as being the moſt ſimple and eaſy of all others, I laid it aſide at laſt, not being able to ſurmount, at that time, all the Difficulties* *which occur'd. But I was ſatisfied with having reduced the Conick Sections to a Plane, which I called* Plane Conicks ; *and I applied to thoſe* Plane Sections *the ſame Demonſtrations which I had before made for the* Solid. *And I can ſay, that that Work had the good Fortune to have the Approbation of many learned Geometricians.*

But although it be very advantagious to pleaſe the Learned, yet we ought by no means to make that the ſolid Object and ultimate End of our Studies, and to neglect the Inſtruction of thoſe that are deſirous to underſtand Things of this Nature ; and I think they ought to be contented when they are ſhewed ſeveral Ways of doing the ſame thing ; for then every one may make choice of that which he likes beſt.

Without ſtaying to give you an Enumeration of ſuch Authors as have written well on this Science, or ſhewing you the different Methods of deſcribing theſe Curves uſed by them ; I ſhall only inſiſt on the Difference that there is between my Method, and that of Mr. de Witt, *which hath been juſtly thought the beſt.*

He makes uſe of a particular Method for the Deſcription of each Section, *and which, eſpecially in the* Parabola *and* Hyperbola, *is embaraſſed with many Lines,* *interſecting one another in certain Angles; by which means it is difficult at firſt for the Learner to get a clear and plain Idea of the Formation of theſe Curves; whereas the Deſcription that I uſe, and which is taken from the principal Properties of the* Foci, *hath nothing for its Rule but one ſingle Line, which in the Ellipſis is equal to the Sum, in the* Hyperbola *is equal to the Difference of two other Lines drawn from the* Foci *to a Point in the Curve deſcribed; and in the* Parabola *the Sum and Difference are found together; for beſides its* Focus, *which is determined in the* Axis, *if you ſuppoſe alſo another in the* Axis *at an infinite Diſtance within the* Parabola *below, it is evident that the Sum of theſe Lines, which ſhall be drawn from any Point in the Curve of the* Parabola, *to thoſe two* Foci, *ſhall be equal to one and the ſame Line drawn from the infinitely diſtant* Focus *to a Line, which being perpendicular to the* Axis, *ſhall meet it in a Point, which is as far diſtant from the Vertex of the* Parabola *as the determined* Focus.

And if you ſuppoſe another Focus *infinitely diſtant in the* Axis *above, without the* Parabola, *it is alſo evident, that the Difference of two Lines drawn from any*

Point in the Curve of the Parabola, *to that indetermined* Focus, *and to the other determined one ſhall be equal always to one and the ſame Line.*

Theſe are the Properties, and theſe are of very great uſe in Geometry, eſpecially in Catoptricks, Dioptricks *and* Aſtronomy.

I believe Mr. de Witt *would not have uſed ſo compounded a Method for the Deſcription of theſe Curves, had he not thought that the Demonſtrations would have been more eaſy and ſimple on that Account; which he could not bring to paſs in the Ellipſis, when he endeavoured to demonſtrate the Properties of the Diameters. The Demonſtration which I have given you, hath ſome Conformity with that of the Antients, but it is much more ſimple and Plain.*

This Book contains all the Elementary Properties of the Conick Sections; and there is no more Geometry required, than to underſtand thoroughly the firſt ſix Books of Euclid's *Elements, or the Subſtance of what is contained in them, as it hath been demonſtrated divers Ways by many learned Geometricians. I have by no means changed the Names which* Apollonius *gave to the Curves, both becauſe Cuſtom hath*

hath familiarized them to us, and alſo becauſe I have demonſtrated in their Order, the Properties on which they depend. The Parabola *takes its Name from the Equality that there is between the Squares of the Ordinates to the Diameter, and the Rectangles under the Parameter and the* Abſciſſæ; *as I have demonſtrated in the 1ſt Corollary of the 1ſt Propoſition, and in the 14th of the* Parabola.

The Ellipſis *is ſo called, becauſe the Rectangle under the Parameter and the* Abſciſſæ, *or the Parts of the Diameter, intercepted between the Vertex and the ſevelar Ordinates, are leſſer than the Squares of ſuch Ordinates, by a Rectangle ſimilar to thoſe Rectangles. As I have ſhewed in the 5th and 18th Propoſitions about the* Ellipſis. *In the* Hyperbola, *the aforeſaid Rectangle under the Parameter and the* Abſciſſæ, *exceeds the Squares of the ſeveral Ordinates, by a ſimilar Rectangle,* &c. *as I have demonſtrated in the 3d and 22d Propoſitions of the* Hyperbola.

You will find in the Treatiſe the moſt uſeful Part of the firſt Book of Apollonius; *the Properties of the Aſſymptotes, which he delivers in his ſecond Book, and the firſt Propoſitions of the Third, and thoſe about the* Foci; *which are the moſt conſiderable.*

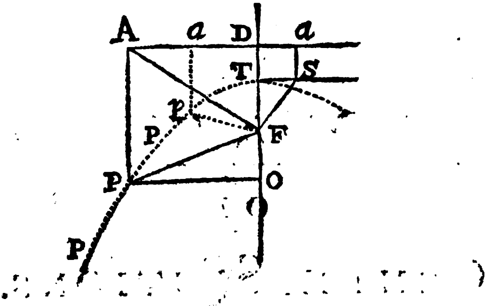
As to the Proportions which I make uſe of, I cite only Compoſition, Diviſion *and Equality of* Ratios; *becauſe I ſuppoſe my Reader knows how to change* alternately, *or to* invert *or* convert *the Terms of four Proportionals,* &c.

A
TREATISE
OF
CONICK SECTIONS.

PARABOLA.

The GENERATION *of the* PARABOLA.

If a right Line, as AD, *be drawn upon a Plane, and a Point, as* F, *be taken without this Line; I ſay, an infinite Number of Points, as* P, P, P, *may be found in ſuch an Order, that the Line* FP, *drawn from the Point* F *to each Point* P, *ſhall be equal to* PA, *drawn from the ſame Point P perpendicular to A D.*



FROM any Point at Pleaſure in the Line A D (ſuppoſe A) having drawn A P perpendicular to A D, and likewiſe the Line F A; make the Angle A F P equal to the Angle F A P, and the Point P will be one of the Points required: And after the ſame Manner an infinite Number of others may be found.

COROLLARY.

THROUGH the Point F having drawn the Line F D perpendicular to A D; it is evident that P F (the Line which paſſes through the Points P p) biſſects F D in T, and that the Line P P T increaſeth in-

infinitely, and likewiſe runs off infinitely from the Line FD produced down towards the Part F.

DEFINITION I.

THE Curve Line PPT, formed by being drawn through the Points PP, is call'd a *Parabola.*

DEFINITION II.

THE Point T is called the *Vertex*, and the Point F the *Focus* of that *Parabola.*

DEFINITION III.

THE Line DFO the *Axis*; *and any part of it, as* OT, *lying between the* Vertex *and any Point, as* O, *is called the* Abſciſſa.

DEFINITION IV.

THE Line PO drawn from any of the Points P of the *Parabola*, perpendicular to the *Axis*, is called an *Ordinate* to the *Axis.*

DEFINITION V.

ALL the Lines within the *Parabola*, which are drawn parallel to the *Axis*, are called *Diameters.*

DEFINITION VI.

A RIGHT Line, which meets the *Parabola* only in one Point, and which does not cut it, is called a *Tangent* to it in that Point.

PROP. I.

The preceding Generation of the Parabola *being ſuppoſed; I ſay, the Square of any Ordinate, as* PO, *is equal to a Rectangle under twice the Line* FD *multiplied into* TO; *that Part of the* Axis, *which is contain'd between the* Vertex T *and* O, *the Point where it meets the* Ordinate.

LET $TD = a$; then will FD be $= 2a$; and let $FO = b$; $DO = AP = FP$ (*by the Generation of the* Parabola) and conſequently $DOq = FPq$; but $FPq = POq + bb$, and $DOq = 4aa + 4ab + bb$; therefore $POq + bb = 4aa + 4ab + bb$, and taking away bb from each Side of the Equation, there remains $POq = 4aa + 4ab$; but $4aa + 4ab = 2FD \times OT$; therefore $POq = 2FD \times OT$. *W. W. D.*

COROL.

COROLLARY I.

IT is evident from the foregoing *Prop.* that the Square of each Ordinate is equal to a Rectangle, under a Line, which is the Double of F D, *and conſequently an invariable Quantity*, and T O the *Abſciſſa*, or that Part of the *Axis* intercepted between the *Vertex* T and O, the Foot of the Ordinate.

DEFINITION.

THE Line equal to 2 FD is called the *Parameter* of the *Axis*, and by ſome the *Latus Rectum*.

COROLLARY II.

IT is evident that the *Focus* is diſtant from the Extremity of the *Axis*, or *Vertex*, $\frac{1}{4}$ Part of the *Parameter*.

COROLLARY III.

IT is likewiſe evident that the Square of the Ordinates are to each other, as their reſpective *Abſciſſæ*, or the Parts of the *Axis* comprehended between the *Vertex* and thoſe Points where each of them meets its reſpective Ordinate; for all of them are equal to Rectangles whoſe Baſes are thoſe

Abſciſſæs, and whoſe common Altitude is the Parameter ; *but Rectangles of the ſame or equal Altitudes are as their Baſes.*

PROP. II.

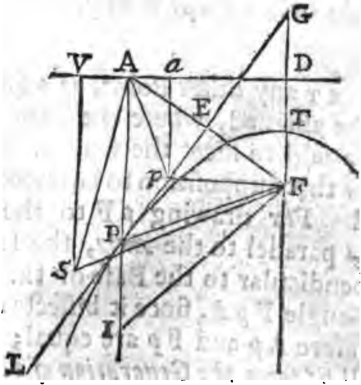
The right Line TS, *drawn through the Point* T, *parallel* to PO, *touches the* Parabola *in the ſame Point* T.

1*ſt*. If it be poſſible, let the Line TS (*See Fig. in Pag.* 2.) meet the *Parabola* in any other Point as S ; then having drawn S*a* parallel to the *Axis*, FS = S*a* (*by the Generation*) wherefore FS = FT (becauſe FT, TD, and S*a* are equal) which is abſurd : For in the Right-angled Triangle FTS, the Side FS, which ſubtends the Right Angle, is greater than either of the other Sides.

2*dly*. It is ſtill leſs poſſible that TS ſhould ever fall within the *Parabola*, for then the *Parabola* would fall between D*a* and TS, which is impoſſible *from the Generation* : Therefore the Proportion is true.

PROP. III.

The ſame Things being ſuppoſed as before; I ſay, that a Diameter, as P I, *can meet the* Parabola *only in one Point* P.



Suppose the Diameter meet the *Parabola* in another Point, as I, the Line I F will be = I A *(from the Generation)* but P F = P A for the ſame Reaſon; wherefore P F + P I will be = I F, which is abſurd; for any two Sides of a Triangle, taken together, are greater than the third; wherefore the Propoſition is true.

PROP. IV.

The ſame Things being ſtill ſuppoſed as above, draw the Line F A, *and having biſſected it in* E, *draw* P E. *I ſay, the right Line* P E *touches the* Parabola *in the Point* P *only.*

1. LET any other Point, as *p* (*See Fig. p.* 7) be aſſigned, where the Tangent P E may be ſaid to meet the Curve: It is eaſy to ſhew that Suppoſition to be impoſſible or abſurd. For drawing *p* F to the *Focus*, and *p a* parallel to the *Axis*, the Line *p* E is perpendicular to the Baſe of the Iſoſceles Triangle F *p* A, ſince it biſſects it in E: Wherefore A *p* and F *p* are equal; but *a p* = to F *p* (*from the Generation of the Parabola*) therefore *a p* will be = A *p*, which is abſurd: For the Angle *p a* A is a right one; therefore the Line P E does not meet the *Parabola* in the Point *p*.

2. AGAIN, if it be poſſible, let the *Parabola* below the Point P, fall without the Line E P L, with reſpect to the Axis, that is, let the Part P L of the Line E P L be always within the *Parabola*. From any one of

the Points, as S, of the ſuppoſed Parabola, lying without the Line L P, having drawn S F to the *Focus*, and S V parallel to the *Axis*; (*by the Generation of the Parabola*) S F = S V; but S A is greater than S V, the Angle S V A being a right one; therefore S A is greater than S F, which is abſurd: For the Line E P L biſſects A F, and is perpendicular to it, *and likewiſe paſſes through the Point* S; wherefore S A and S F will be equal. Therefore it is evident, that S is not one of the Points of the *Parabola*, and conſequently that the Propoſition is true.

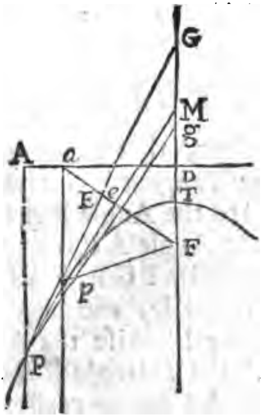
COROLLARY.

It is evident, that the Angle F G E, made by the Tangent P G and the *Axis* F G, is equal to the Angle F A D, which is equal to the Angle A P E or F P E; for the two Triangles F G E, F A D are right-angled at E and D, and have the Angle at F common; likewiſe the Angle at the Point A of the Triangle P A E is equal to the Angle A F D, by reaſon of the parallel Lines P A, F D; and the Triangle F P E is equal and ſimilar to the Triangle A P E. It is alſo manifeſt, that the Triangle P F G is an Iſoſceles one.

PROP. V.

There can be but one Line, as PEG, *which touches the* Parabola *in the ſame Point* P, *and doth not cut it.*

SUPPOSE it poſſible for another Tangent, as Pg, to touch it in the ſame Point. From the *Focus* F having drawn F*a* perpendicular to Pg, F*a* will meet AD in ſome Point, as *a*: For the Tangent Pg al-



ways meets the *Axis*, and can never be parallel to it; for then it would be a Diameter, and would paſs within the *Parabo-*

la: Having drawn *a p* parallel to the *Axis*, and ſo that it meets the *Parabola* in *p*, *p* F will be equal to *p a* (*by the Generation of the Parabola*) and F *a* being biſſected in *e*, the right Line *p e* M will touch the *Parabola* in *p*, and will be parallel to P g, *by the precedent Propoſition, being each of them perpendicular to* F *a*: But the Tangent P E meets the other Tangent *p e* without the *Parabola*, ſince both of 'em are Tangents; wherefore P g parallel to *p e* will paſs within the *Parabola*, which is abſurd; for it was ſuppoſed a Tangent, and conſequently ought to be without: Therefore the Propoſition is true.

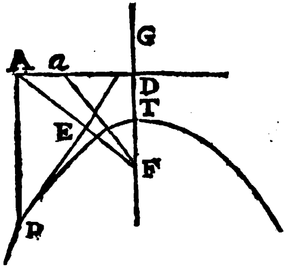
COROLLARY.

'TIS evident from what has been demonſtrated, that all the Tangents meet both the *Axis*, and alſo the Diameters, and that they likewiſe meet one another, ſince (*by the 4th Propoſition*) they are not parallel.

PROP. VI.

To find a Tangent, which on the ſame Side with the Parabola *and the Tangent, makes an Angle with the* Axis *leſs than any given Angle; provided the Angle given be not greater than a Right one.*

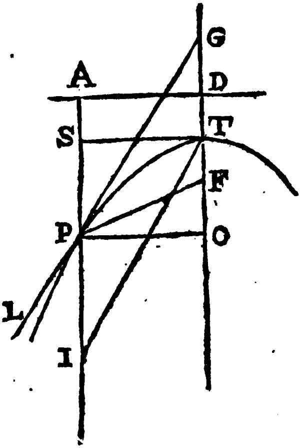
LET the Line F *a* be drawn ſo, that the Angle F *a* D may be equal to the Angle given; then take the Point A, on the other Side of *a*, with regard to the *Axis*: I ſay, it is evident, that the Angle F A D will be leſs than the given Angle F *a* D; and (by *Cor. Prop.* iv.) the Line P E G drawn



through the Point P (where the Line A P parallel to the *Axis* meets the *Parabola*) and the Point E (which biſſects F A) will be a Tangent to the *Parabola* in the Point P; and will, with the *Axis*, make the Angle P G F equal to the Angle F A D, which is leſs than the given Angle F *a* D, which was to be done.

PROP VII.

Suppoſing the ſame Things as before; I ſay, that the Part of the Axis T G, *comprehended between the Extremity* T *and the Point* G, *where the Tangent meets it, is equal to* T O, *the Part contained between the ſame Extremity* T, *and* O *that Point where the Ordinate* P O *drawn from* P, *the Point of Contact of that Tangent, meets the Axis.*



By *Cor. Prop.* iv. the Triangle P G F is an Iſoſceles; therefore F G is = F P = P A (*from the Generation of the Parabola*) or DO, becauſe PO is parallel to AD; and if from the equal Lines FG, DO be taken the equal Lines FT, DT, the remaining Lines, TG, TO will be equal. *W. W. D.*

COROLLARY.

If the Tangent TS be drawn thro' the Point T, and produc'd till it meet the Diameter API; and if the Line TI be drawn parallel to the Tangent PG; it follows that PS and PI are equal; for PS is = TO, and PI = TG.

PROP. VIII.

The ſame Things remaining as before, the Angle IPL *made by the Tangent* PL, *and the Diameter* PI *drawn thro'* P, *the Point of Contact is equal to the Angle* FPG, *contain'd between the Tangent* LPG *and the Line* FP, *drawn from the Focus* F *to the Point of Contact* P.

By *Cor. Prop.* iv. (*See Fig. in p.* 13.) the Angle FPG is equal to the Angle GPA, which is equal to the Angle LPI; wherefore the Angle IPL is equal to the Angle FPG. *W. W. D.*

CO-

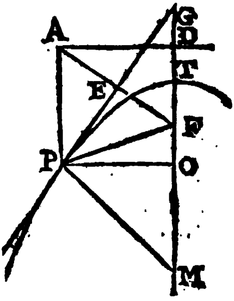
COROLLARY.

It is alſo apparent that the Angle L P F is equal to G P I, by adding the common Angle F P I to the two equal Angles I P L, and F P G.

PROP. IX.

If through the Point of Contact P, P M *be drawn perpendicular to the Tangent, and continu'd till it meet the Axis in* M; *I ſay the Line* O M *contain'd between the Foot of the Ordinate* O, *and the Point* M, *is equal to half Parameter of the Axis.*

The Line P G is perpendicular to F A, (*by Prop.* iv.) wherefore A F and P M are



parallel; but the Triangles A D F and P O M are equal and ſimilar by reaſon of the Parallels P A, D M and A D, P O; wherefore O M is equal to D F, which is equal $\frac{1}{2}$ the Parameter *(by the Definition of the Parameter.)* W. W. D.

PROP. X.

In the Parabola T E P, *whoſe* Axis *is* T O, *draw the Tangent* P H *to the Point* P, *meeting the Axis in* H, *and another Tangent, as* T B, *to the Vertex* T, *meeting another Diameter, as* D P B, *produced to* B; *then will the Triangle* T A H, *made by the two Tangents and the Axis, be equal and ſimilar to the Triangle* B A P, *made by the ſame Tangents, and the Diameter* B P *produced.*

HAVING drawn P O (*See Fig. in Pag.* 18.) at right Angles to the Axis, T O is = T H *(by the ſeventh Prop.)* but B P is = T O; therefore P B = T H: Theſe Lines B P and T H are alſo parallel, as likewiſe are B T and P O; wherefore the Triangles T A H and B A P are equal and ſimilar. *W. W. D.*

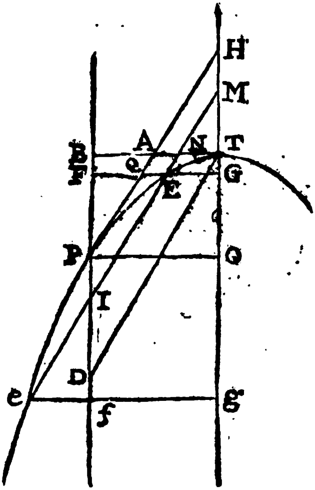
COROLLARY.

HAVING drawn T D parallel to P H, if each of the equal Triangles be added to the Trapezium T D P A, the Triangle T D B will be equal to the Parallelogram T D P H, which is equal to the rectangled Parallelogram T O P B, becauſe they have equal Baſes T O, T H, and are between the ſame Parallels; but the Rectangle T O P B is yet farther equal to the Triangle P O H, by reaſon of the two equal Triangles A P B and A T H.

PROP. XI.

The ſame Things being ſuppoſed as before; If two Lines, F G and e M, *be drawn through any Point of the Parabola, as* B, *parallel to the two Tangents* P A H *and* B A T; *the Triangle* E G M *is equal to the Rectangle* G T B F.

From the Nature of parallel Lines, the



Triangle P O H. Triangle E G M : : P O*q* : E G*q*; but (*by Prop.* i. *Cor.* iii.) P O*q*, E G*q* : :

EG*q* :: TO. TG : And TO. TG :: Rectangle TOPB. Rectangle TGFB: Therefore (*by Proportion of Equality*) the Triangle POH. Triangle EGM :: Rectangle TOPB. Rectangle TGFB; but the Triangle POH is equal to the Rectangle TOPB (*by Cor. Prop. precedent*;) therefore the Triangle EGM is equal to the Rectangle TGFB. *W. W. D.*

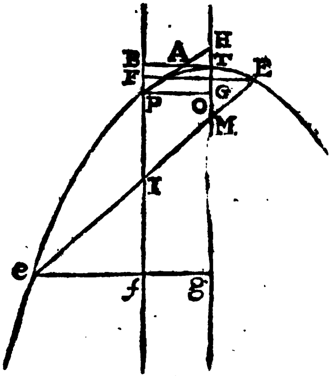
After the ſame manner the Triangle *eg*M may be demonſtrated equal to the Rectangle T*gf*B.

PROP. XII.

The Triangle EFI *is equal to the Parallelogram* PIMH.

If the Point E be between P and T (*See Fig. in the foregoing Page*) the Triangle POH (by *Cor. Prop.* x.) is equal to the Rectangle POTB; from which if you take equal Things, *viz.* if from the Triangle POH you take the Triangle MEG, and from the Rectangle POTB, the Rectangle TGFB equal to the Triangle MEG: (*by Prop.* xi.) And again, if from each you take the Quadrilateral Figure PQGO, there remains the Quadrilateral Figure EQHM equal to the Triangle QFP; to each of which, if the common Quadrilateral Figure PIEQ be added, the Parallelogram PIHM will be equal to the Triangle EFI. *W. W. D*

But if the Point E be on the other Hand of the Point T, with regard to P, the Triangle ABP *(by Prop.* x.*)* is equal to the Triangle ATH, and adding the



common Figure PIMTA to each of them, the Parellelogram PIMH will be equal to the Trapezium IMTB, from which Trapezium if the Rectangle GTBF be taken, and to the remaining Figure FGMI be added to the Triangle GEM, which is equal to the Rectangle GTBF (*by Prop.* xi.) then will the Triangle EFI be equal to the Parallelogram PIMH.

Lastly, If the Point E be on the other Side of P, as in *e*; the Triangle TAH is equal to the Triangle ABP; (*by Prop.* x.) and if the Figure T*gf* PA be added to each, the Quadrilateral Figure H*gf*P will be equal to the Rectangle T*gf*B, which is equal to the Triangle *ge*M (*by the precedent.*) Then if the common Quadrilateral Figure M*gf*I be taken from the Quadrilateral Figure H*gf*P, and the Triangle M*ge*, which is equal to it; there remains the Parallelogram PIMH equal to the Triangle *ef*I. *W. W. D.*

PROP. XIII.

If from a Point E *in the* Parabola (*Fig.* P. 20, 22.) *be drawn the Right Line* Ee *parallel to a Tangent, as* PH; *then will the Right Line* Ee *meet the* Parabola *in another Point, as* e; *and likewise be bissected in* I *by the* Diameter PI, *drawn through* P, *the Point of Contact.*

If BT be the Tangent, the First Part of the *Prop.* is evident *from the Generation of the Parabola:* But if another Tangent be drawn, as PH, a Tangent (*by Prop.* vi.)

may be found, which ſhall make an Angle with the *Axis*, leſs than the Angle GHP, and on the ſame Side of the *Axis*; wherefore this Tangent will meet the Tangent PH on one Side of P, and the Tangent TA on the other; and the Line E*e* being parallel to PH, will alſo meet the two Tangents either without the *Parabola* or upon it, on each Side the Diameter PI, and conſequently the *Parabola* in two Points, as E, *e* (which is the firſt Part of the *Prop*)

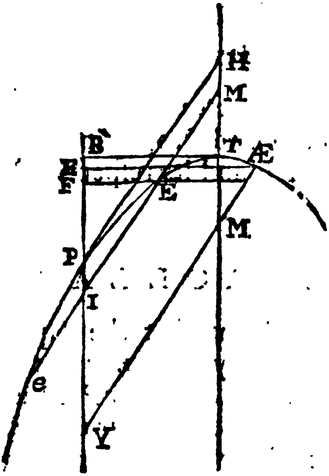
THE Triangle EFI (*by Prop. præced.*) is equal to the Parallelogram PIMH, which is alſo equal to the Triangle *ef*I; wherefore the two Triangles are equal: But they are likewiſe ſimilar (by reaſon of the parallel Lines which form them) therefore EI is equal to *e*I, *W. W. D.*

DEFINITION.

THOSE Lines are called *Ordinates* to a *Diameter* which are drawn parallel to a Line touching the *Parabola* in the *Vertex* of that *Diameter*; as thoſe Lines are *Ordinates* to the *Axis* which are Perpendicular to it.

PROP. XIV.

The Squares of the Ordinates, *as* EI, ÆY, *belonging to one and the ſame* Diameter, *as* PI, *are to one another as the Parts of this* Diameter PI, PY *compriz'd between the* Vertex P, *and the Points where it meets the* Ordinates I *and* Y.



For (the Triangles EFI, and ÆFY being ſimilar) the Triangle EFI is to the Triangle ÆFY as the Square of EI to the Square of ÆY; but (*by Prop.* xii.) the Triangle EFI is equal to the Parallelogram PIMH; and the Triangle ÆFY is equal to the Parallelogram PYMH; and theſe Parallelograms are to one another as PI to PY; wherefore the Square of EI is to the Square of ÆY, as PI to PY. *W.W.D.*

'Tis the ſame Thing if the *Ordinates* be taken on the other Side the *Diameter*, as *e*I; for they are equal to the former. (*By Prop.* xiii.)

Definition of the Parameter.

The *Parameter* belonging to any *Diameter* is a Third Proportional to any *Abſciſſa*, as PI, and its reſpective *Ordinate*, as IE.

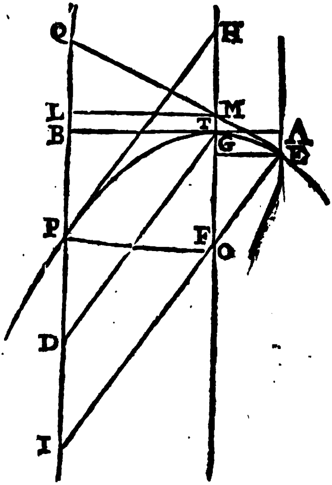
COROLLARY.

It is evident that the Square of an *Ordinate* to any *Diameter* (as ÆY is an *Ordinate* to the *Diameter* PY) is equal to a Rectangle under the *Parameter*, and PY that Part of the *Diameter* which is contained between the *Vertex* P, and Y the Point where the *Ordinate* meets the *Diameter*.

PROP. XV.

If a Tangent, as EQ, *be drawn thro' the Point* E *of a* Parabola, *meeting any* Diameter, *as* PQ *in* Q; *and if thro' the ſame Point* E, *be drawn an* Ordinate *to this* Diameter, *I ſay,* PQ *and* PI *are equal.*

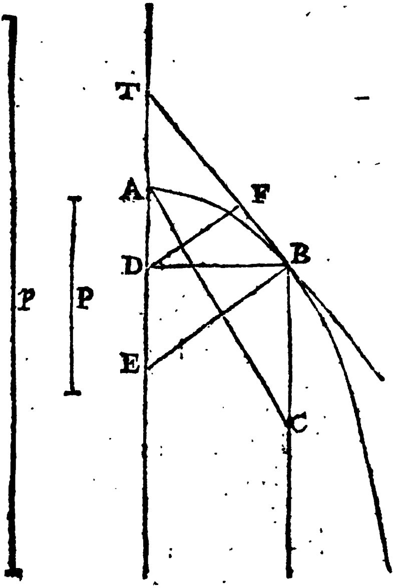
DRAW the *Axis* TF, and BT a Tangent in the Point T, and PH a Tangent in P, which PH will be parallel to EI, (by



Prop. xiii.) to which E I let T D be drawn parallel, as likewiſe M L, E G, P O, parallel to B A; then (by *Cor.* 3. *Prop.* 1.) the Square of P O, or B T, which is equal to it, is to the Square of E G as T O to T G, or as their Doubles B D to M G. And becauſe of the Parallel Lines, the Square of B T is to the Square of E G as the Square of B D to the Square of G F; therefore the Square of B D is to the Square of G F as B D is to M G, and conſequently G F is a mean Proportional between B D and M G. In like Manner the Square of B T, or the Square of L M, is to the Square of G E as the Square of Q L to the Square of M G; wherefore the Square of Q L is to the Square of M G as B D to M G; Q L therefore is a mean Proportional between the ſame B D, M G; and conſequently G F and Q L are equal; and if to theſe two equal Lines be added the equal Lines L B and T G, as likewiſe the equal Lines B P and T H, the two Sums will be equal; that is, P Q will be equal to F H, which is equal to P I (*from the Nature of parallel Lines.*) W. W. D.

PROP. XVI.

" *In the* Parabola AFB, *whoſe* Axis *is*
" AE, *and* Diameter BC; *from the*
" *Extremity* B *of the* Diameter BC,
" *draw* BD *an Ordinate to the* Axis
" AE. *I ſay, the* Parameter *of the*
" Diameter BC, *exceeds the* Parame-
" ter *of the* Axis AE, *by Quadruple*
" *the intercepted* Axis AD.



“ HAVING drawn BT touching the *Parabola* in B, and meeting the *Axis* in T; BE perpendicular to the Tangent in B; DF parallel to BE; and AC parallel to BT, which AC will be an *Ordinate* to the *Diameter* BC (*by Def. Prop.* xiii.) and conſequently equal to TB (and $BC = AT$ from parallel Lines) which is equal to AD; (*by Prop.* vii.): But DE is equal to the Semiparameter of the *Axis* (*by Prop.* ix.)

“ LET the Right Line P be equal to the *Parameter* of the *Axis*, and *p* equal to the *Parameter* of the *Diameter* BC. Then (*Prop.* i. and *Prop.* xiv. *Cor.*) ACq is to BDq, as the Rectangle $BC \times p$ is to the Rectangle AD equal to $BC \times P$; But theſe Rectangles having equal Altitudes, are to one another as *p* to P; and, from the Similarity of Triangles, the Square of BT is to the Square of BD, as BT to BF; and BT is to BF as ET to ED; but ET is equal to DT, or DA twice taken, together with DE, which DE is equal to $\frac{1}{2}$ P; then ET is to ED as 2 ET to 2 ED; but 2 ET is equal to $4DA + P$, and $2ED = P$; therefore ET is to ED as $4DA + P$ is to P, (and *ex Equo*) *p* will be to P as $4DA + P$ is to P;

“ conſequently, p is $=$ to $4DA + P$.
“ *W. W. D.*

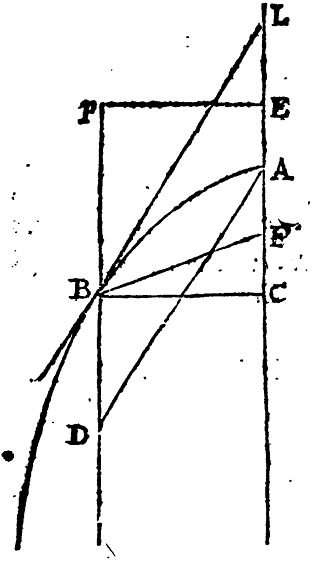
COROLLARY I.

“ IT is evident (*from this Propoſition*)
“ that the *Parameters* of thoſe *Diameters*
“ which are farther diſtant from the *Axis*,
“ are greater than the *Parameters* of
“ thoſe *Diameters* which are nearer to it:
“ For the Square of B D is to the
“ Square of A C, as the *Parameter* of the
“ *Axis* A E to the *Parameter* of the *Dia-*
“ *meter* B C.

PROP. XVII.

“ *In the* Parabola B A, *whoſe* Axis *is*
“ A C, *draw any* Diameter, *as* B D,
“ *and* F B *from the* Focus F *to* B, *the*
“ *Extremity of the* Diameter B D.
“ *And the Right Line* F B *will be*
“ *equal to* $\frac{1}{4}$ *of the* Parameter *of that*
“ Diameter.

“ AE is equal to AF (*by the Conſtruction of the* Parabola) which is equal to $\frac{1}{4}$ of the *Parameter* of the *Axis*, and (*by the laſt Propoſition*) the *Parameter* of the *Diameter* BD exceeds the *Parameter* of the *Axis* by 4 times AC; but AE is equal to $\frac{1}{4}$ of the *Parameter* of the *Axis*; wherefore CE is equal to $\frac{1}{4}$ of the *Parameter* of the *Diameter* BD; but CE is equal



“ to B*p*, which is equal to BF (*by the*
“ *Generation of the* Parabola) therefore
“ BF is equal to $\frac{1}{4}$ of the *Parameter* be-
“ longing to the *Diameter* BD. *W. W. D.*

COROLLARY.

“ HENCE we have a new and very
“ eaſy Way of determining the *Parame-*
“ *ter* of any *Diameter*; for ’tis always
“ Quadruple of a Line drawn from the
“ Extremity of that *Diameter* to the
“ *Focus* of the *Parabola*.

A TREATISE OF CONICK SECTIONS.

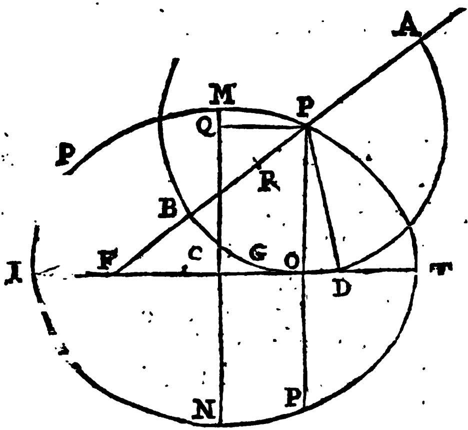
PART II.

The *ELLIPSIS.*

Of the GENERATION *of the* ELLIPSIS.

IF upon a Plane be drawn the Right Line IT, and biſſected in C, and the Points F and D be taken at equal Diſtances from C: I ſay, as many

Points as you pleaſe of P, may be found



in ſuch an Order that two Lines, P F and P D, drawn from that Point P to the two Points F and D, being join'd together, will be equal to the Line I T.

Divide I T any how into two Parts, and make one of them the Radius of a Circle, whoſe Centre let be F, and the other the Radius of a Circle, whoſe Centre let be D; and with theſe Radii, and upon theſe Centres, deſcribe two Circles, which will interſect one another in two Points, as P, P, one of which is above, and the other below the Line I T, and at equal Diſtances from it; and the Line P O P which joins the two Points P, P, will be perpendicular to

IT, and after the ſame Manner an infinite Number of Points may be found, as P; *which was to be done.*

COROLLARY.

It is evident, that a Line paſſing thro' the Points P, P, will paſs thro' I and T, and that the Line PTPI will include Space; and likewiſe that all the Lines, as POP, which are perpendicular to IT, and terminated by the Curve Line P, P, on each Side of the Line IT, will be biſſected in O by the ſame Line IT.

DEFINITION.

1. The Curve Line PTPI is called an *Ellipſis.*

2. The Point C, the *Centre* of the *Ellipſis.*

3. The Right Line IT, the *Tranſverſe Axis.*

4. The Right Line NCM, Perpendicular to IT, paſſing thro' the Centre, and terminated by the *Ellipſis*, is called the *Conjugate Axis.*

5. The Points F and D, are called the *Foci.*

6. Right Lines drawn from the Points in the *Ellipſis* perpendicular to the

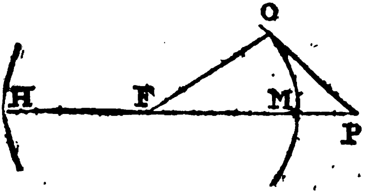
Axes, are called *Ordinates to the Axes*, as PO is an Ordinate to the Axis IT.

7. All the Right Lines which paſs thro' the Centre C, and are terminated at each End by the *Ellipſis*, are called *Diameters*.

8. A Right Line, which meets the *Ellipſis* only in one Point, is call'd a *Tangent* to the *Ellipſis* in that Point.

LEMMA.

In every right-angled Triangle, as FOP, the Rectangle made by multiplying PH, the Sum of the Hypothenuſe FP, and one Side, as FO, into MP their Difference, is equal to the Square of the other Side PO.



From F, as a Centre, and with the Radius FO, having deſcribed the Circle MOH, and drawn PF to H, the Side PO touches the Circle in O, becauſe the Angle at O is right; wherefore the Rectangle PM multiply'd into PH, is equal to the Square of PO. *W. W. D.*

PROP.

PROP. I.

The Ellipſis *being formed according to the precedent Method; the Square of any* Ordinate *to the* Tranſverſe Axis, *as* PO, *is to the Rectangle* IOT, *under the Parts of the* Tranſverſe Axis *each way from the* Ordinate, *as the Rectangle* IFT *is to the Square of* IC *or* CT. (*Fig. in Page* 33.)

Having prolonged FP to A, make FA equal to IT, and biſſect it in R; from the Point P, as a Centre, and with the Radius PD or PA equal to it, deſcribe the Circle ADB, which will meet AF in B, and IT in G, if the Points D and O are not coincident; for ſhould they be coincident, the three Points D, O, G would coincide, and the Circle would touch the Line IT in O. But ſuppoſing firſt, that the three Points D, O, G are not coincident; from the Nature of the Circle ADB, the Rectangle FA×FB is equal to the Rectangle FD×FG; wherefore FA is to FD as FG to FB; and their Halves FR or CT is to CD, as CO is to RP; and (*by Compoſition*) CT is to $CT + CD$ as CO is to $CO + RP$; conſequently CT is to CO as $CT + CD$ is to $CO + RP$; and farther (*by Compoſition*) CT is to $CT + CO$ (which added together, are equal to IO) as $CT + CD$ (which added together, are equal to ID) is to $CT + CD + CO + RP$. But $CT + CD + CO + RP$, or its Equal $FR + RP + FO + CO$ is equal to the Sum of FP and FO; wherefore CT is to IO as ID is to $FP + FO$.

FC ∧

In like manner, reaſſuming that Proportion above of CT to CD, as CO to RP, and, *by Diviſion*, CT is to $CT - CD$, as CO to $CO - RP$; and conſequently, CT is to CO as $CT - CD$ is to $CO - RP$; and *farther by Diviſion*, CT is to $CT - CO$ (which is equal to OT) as $CT - CD$ (which is equal to DT) is to $CT - CD - CO + RP$. But $CT - CD - CO + RP$, or its Equal $FR + RP - FC - CO$, is equal to the Difference of FP and FO; wherefore CT is to OT as DT to the Difference of FP and FO.

Now if the Proportion found above at the End of the laſt Article ſave one, and that which we found l[illegible] be multiply'd one into another, Ant[illegible]nt into Antecedent, and Conſequent [illegible] Conſequent, we ſhall have the Square of CT to the Rectangle IOT as the Rectangle IDT to the Rectangle $FP + FO \times FP - FO$ equal (*by the preceeding* Lemma) to the Square of PO. *W.W.D.*

If the Points D, G, O coincide, the Demonſtration will ſtill be the ſame; for FG, FO, and FD will be equal. Which cannot make any Alteration.

PROP. II.

Retaining the ſame Figure; the Rectangle IDT *is equal to the Square of* CM, *which is half of the* Conjugate Axis.

Suppose MC be an *Ordinate*, then (*by the foregoing Prop.*) the Square of CT is to the Rectangle ICT, *i. e.* the Square of CT, as the Rectangle IDT, is to the Square of MC. Therefore the Square of MC is equal to the Rectangle IDT or IFT, which is equal to it. *W. W. D.*

PROP. III.

Still retaining the ſame Figure. The Square of the Conjugate Axis NM, *is the Square of the Tranſverſe* Axis IT, *as the Square of* PO *an* Ordinate *to the* Tranſverſe, *to the Rectangle* IOT.

THE Square of CT (*Prop.* i.) is to the Rectangle IDT, which is equal (*by the preceding Prop.*) to the Square of CM, as the Rectangle IOT to the Square of PO; but the Square of IT is quadruple the Square of CT, and the Square of NM, is alſo quadruple the Square of CM; therefore the Square of IT is to the Square of NM as the Rectangle IOT to the Square of PO. *W. W. D.*

PROP. IV.

Still retaining the ſame Figure as above, the Square *of* PQ, *which is an* Ordinate *to the* Conjugate Axis, *is to the Rectangle* NQM, *as the Square of the Tranſverſe* Axis IT, *is to the Square of the* Conjugate Axis NM.

By the firſt and ſecond Propoſition, the Square of CT is to the Square of CM as the Rectangle IOT (which is equal to the Square of CT leſs the Square of CO) is to the Square of PO; and, *by Diviſion*, the Square of CT is to the Square of CT leſs the Square of CT more the Square of CO, as the Square of CM is to the Square of CM leſs the Square of PO; and, by the preceeding Propoſition, the Square of IT is to the Square of NM, as the Square of CO or PQ is to the Rectangle NQM (which is equal to the Square of CM leſs the Square of PO or CQ.) *W. W. D.*

DEFINITION.

A THIRD Proportional to the two *Axes* is call'd the *Parameter* of that *Axis* which ſtands firſt in the Proportion; thus if the *Tranſverſe Axis* IT be to the *Conjugate* NM, as the *Conjugate* NM is to a Right Line YT, this YT is called the *Parameter* of the *Tranſverſe*; but if NM (*See Fig. in Pag* 41) be to IT as IT to another Line, that Line will be the *Parameter* of the Conjugate.

2. THE *Figure* of an *Axis* is a Rectangle under that *Axis* and its *Parameter*, as the Figure of the *Tranſverſe* IT is the Rectangle ITY.

CO-

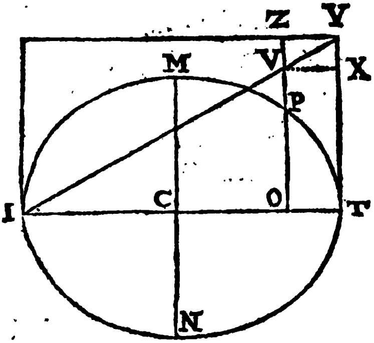
COROLLARY.

IT is evident, that the Square of one of the *Axes* is equal to the *Figure* of the other *Axis.*

PROP. V.

Suppoſing the Ellipſis *conſtructed as above, the Square of* PO, *an* Ordinate *to the* Axis IT, *is equal to the Rectangle* OTXV *under* TY *the* Parameter *belonging to this* Axis, *and* OT *the Part compriſed between the Extremity* T *and* O, *the Point where the* Ordinate *meets the* Axis, *leſs the Rectangle* VXYZ *which is ſimilar, and ſimilary poſited to the Rectangle* IY (*the Figure of* IT.)

THE Square of NM (*by Prop.*iii *and* iv.)



is

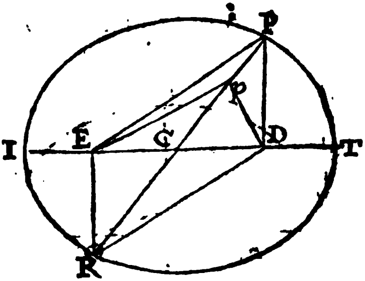
is to the Square of I T, as P O*q* is to the Rectangle I O T; but (*by the Property of the Parameter*) the Square of N M is to the Square of I T, as Y T is to I T; and as Y T is to I T, ſo alſo is V O to I O; or rather the Rectangle V O T to the Rectangle I O T, having T O for their common Altitude, and (*by Proportion of Equality*) the Square of P O is to the Rectangle I O T as the Rectangle V O T X is to the Rectangle I O T: Wherefore the Square of P O is equal to the Rectangle O T X V, which is leſs than the Rectangle O T Y Z by the Rectangle V Y, which V Y is ſimilar and ſimilarly poſited to the Figure I Y. *W. W. D.*

PROP.

PROP. VI.

All the Diameters, as PR, *are biſſected in the Centre* C.

IF it be poſſible for CP to be greater

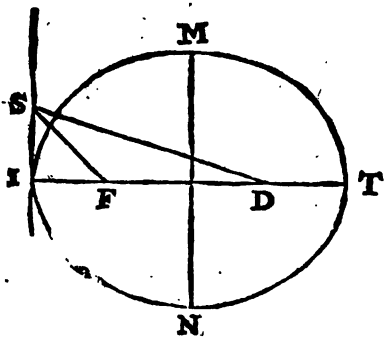


than CR, make C*p* equal to CR; and having drawn to the *Foci* the Lines E*p*, D*p*, ER, DR, and EP, DR, the two Triangles CD*p* and CER will be equal and ſimilar, as likewiſe will the Triangles CDR and CE*p*, for CD, CE, and CR, C*p* are equal; wherefore ER is equal to D*p*, and DR to E*p*; the Sum therefore of E*p*, D*p* will be equal to IT; for the Sum of ER, DR is equal to it (*from the Generation of the* Ellipſis) but the Sum of EP, DP is alſo equal to IT; therefore the Sum E*p*, D*p* is equal to the Sum of EP, DP, which is abſurd. Wherefore the Propoſition is true.

PROP. VII.

A Right Line, as SI, *being perpendicular to the Tranſverſe* Axis, *and meeting its Extremity* I, *touches the* Ellipſis *in that Point.*

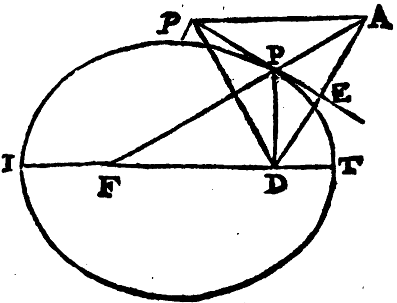
For if SI do not touch the *Ellipſis* in the Point I only, but meet it in ſome other Point, as S; having drawn the Lines FS, DS, to the *Foci*; *from the Generation*, the Lines FS and DS added together will be equal to IT; and becauſe the Triangle FIS is Right-angled at I, FS is greater than FI; wherefore DS muſt be leſs than TF or DI, which is abſurd; for DS ſubtends the Right-Angle I in the Triangle DIS. Therefore the *Propoſition* is true.



P R O P. VIII.

The ſame Things being ſuppoſed as in the firſt Propoſition, *draw the Line* D A, *and biſſect it in* E, *and then the Line* P E *will touch the* Ellipſis *in the Point* P.

If P E do not touch the *Ellipſis* in the

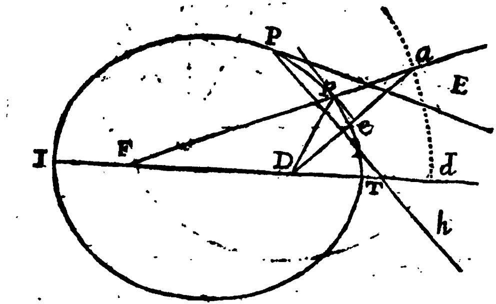


Point P, it will meet it in ſome other Point as *p* (*by the Generation*) F *p*, *p* D added together will be equal to F P and P D added together, which is equal to F A; but *p* D is equal to *p* A, for *p* E is perpendicular to A D, and biſſects it; wherefore F *p* and *p* A together will be equal to F A, which is abſurd; for the two Sides of the Triangle F *p* A can never be equal to the other Side F A; therefore the Line P E touches the *Ellipſis* in the Point P. *W.W.D.*

PROP. IX.

In an Ellipſis, *as* IPT, *there cannot be drawn above one Line, as* PE; *which will touch the* Ellipſis *in the ſame Point* P.

SUPPOSE it be otherwiſe, and that

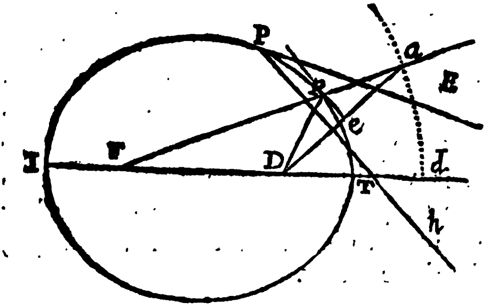


the Line P *b* touch the *Ellipſis* at the Point P: From the Focus D, having drawn D *a* Perpendicular to P *b*, and from the Centre F, and with the Radius F *d* equal to IT, having deſcrib'd the Circle *d a* meeting D *a* in *a*, and having biſſected D *a* in *e*, draw F *a* meeting the *Ellipſis* in any Point as *p*; then draw D *p* and *p e*, which paſſing through *e* the Point of Biſſection of the Line D *a*, will be perpendicular to D *a*; for D *p* and *p a* are equal (*by the Generation of the* Ellipſis) wherefore the Line *p e* will touch the *Ellipſis* (*by the precedent Propoſition*) at the Point *p*; but P *b* is alſo perpendicular to D *a*; wherefore P *b* and *p e* will be parallel; and the Tangents P *b* and *p e* can never meet one another but without the *Ellipſis*; and the *Ellipſis* paſſes through the Points P, *p*; wherefore the Line P *b* parallel to *p e* will neceſſarily meet the Lines F *p* and D *p* within the *Ellipſis*; P *b* therefore paſſes within the *Ellipſis*, and will not touch it as was ſuppoſed: That Line cannot be joined to the Tangent *p e*, for then it would meet the *Ellipſis* in two Points P, *p*; therefore there can be but one Line, which ſhall touch an *Ellipſis*, in the ſame Point.

PROP. X.

The ſame Things being ſuppoſed; The Angles F*p*P, *and* D*pe*, *made by the Tangent pe, and the Lines p*F *and p*D *drawn from p, the Point of Contact to the two* Foci *are equal.*

The Triangle D*pa* is Iſoſcelar, and



pe biſſects the Baſe; wherefore the Angle D*pe* is equal to the Angle *ape*; but the Angle F*p*P is equal to the Angle *ape*; therefore F*p*P is equal to D*pe*. *W.W.D.*

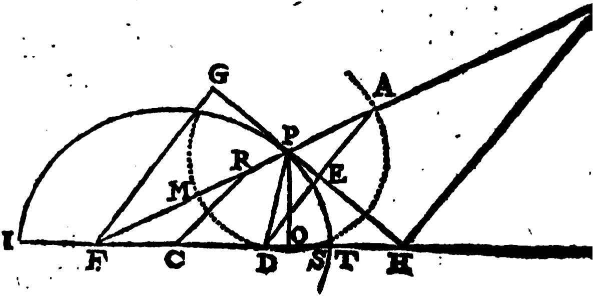
COROLLARY.

IF the common Angle F *p* D be added to the two equal Angles, it is evident, that the Angle *e p* F will be equal to the Angle D *p* P.

PROP. XI.

Suppoſing ſtill the ſame Things, if the Tangent PE *meet the Axis* IT *in* H; *I ſay,* CO *is to* CT, *as* CT *to* CH.

To the Points F, C, H, having drawn



FG, CR, and HV parallel to DA; FA will be biſſected in R, ſeeing FD is biſſected in C: From ſimilar Triangles FP is to AP, as FG is to AE, or its Equal

DE, and FG is to DE as FH to DH; and FH is to DH, as FV to VA; wherefore *ex equo* FP is to PA, as FV to VA; and by *Diviſion* FP — PA (which is equal to twice RP) is to PA, as FV — VA (which is equal to FA) is to VA; but 2RP is to FA, as their Halves, *viz.* RP is to RA; wherefore *ex equo* RP. PA :: RA. VA. By *Compoſition*, RP. RP + PA (= RA) :: RA, RA + VA = RV. Therefore the Lines RP. RA, and RV are continual Proportionals; wherefore the Square of RA is equal to RP × RV.

Now *from the Nature of the Circle*, ASDM, *as in the firſt Propoſition*, which meets the *Tranſverſe* IT in D and S, or in the Point O only, if O and D coincide; then FD will be to FA as FM to FS; and their Halves CD. RA :: RP. CO. But CD. RA :: CH. RV; wherefore *ex equo* CH. RV :: RP. CO, and the Rectangle of the Extreams and Means being equal CH × CO = RP × RV; but in the preceding Article RP × RV was demonſtrated to be equal to the Square of RA, which RA is equal to CT; for (*from the Generation of the* Ellipſis) FA = IT; wherefore CO. CT :: CT. CH. *W. W. D.*

COROLLARY.

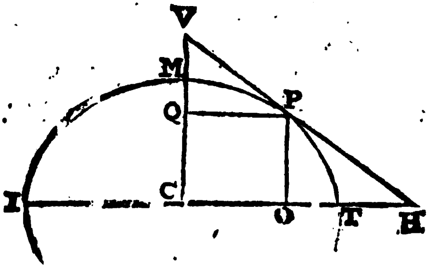
SEEING the Lines CO, CT, CH are in continual Proportion, by inverting the Method of the Demonſtration of the firſt Article of this Propoſition, it will be eaſy to demonſtrate, that IO is to OT as IH to HT; for having found that FV. VA :: FP. PA. 'tis demonſtrable that RP, RA, RV will be in continual Proportion.

THE Line IH is ſaid to be harmonically Divided in the Points O and T.

PROP. XII.

The ſame Things being always ſuppoſed; If the Tangent PE *meet the* Conjugate Axis *in* V, *having drawn* PQ *an* Ordinate *to this* Axis: *I ſay,* CQ *is to* CM, *as* CM *is to* CV.

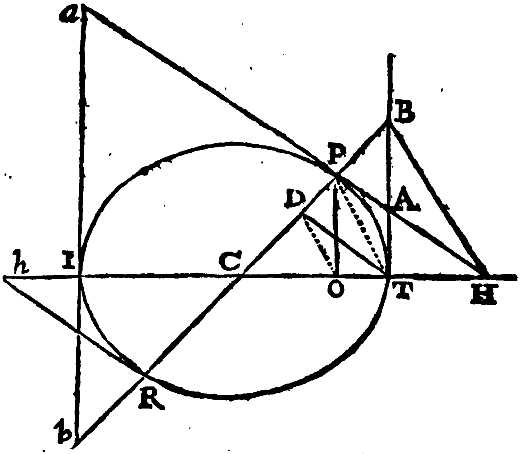
It is evident (*by the foregoing Propoſition*) that the Square of CO is to the Square of CT, as CO is to CH; but (*by Prop.* iv.) CO*q*, or QP*q*. CT*q* :: CM*q* — CQ*q* = Rectangle CQM. CM*q*; and from ſimilar Triangles CO. CH :: VQ. CV; wherefore, *ex equo,* VQ. CV :: CMQ — CQ*q*. MC*q*. By *Diviſion* CV. CV — VQ (= CQ) :: CM*q*.CMQ — CM*q* + CQ*q*, *i. e.* CV. CQ :: CM*q*. CQ*q*; wherefore CQ. CM :: CM, CV. *W. W. D.*



PROP. XIII.

In the Ellipſis IPT, *the* Axis *whereof is* IT, *and* BT *a Tangent to it at the Extremity* T, *and* PA, *another Tangent, meeting the Tangent* BT *in* A, *and the* Axis *in* H. *If through* P, *the Point of Contact of the Tangent* PA, *be drawn the Diameter* CP *meeting the Tangent* TB *in* B; *the Triangles* PAB *and* TAH *are equal.*

FROM the Point T having drawn TD parallel to the Tangent PH, and PO an Ordinate to the *Axis*; *from the Nature of Parallels*, CD. CP :: CT. CH. But (*by*



Prop. xi.) CT. CH :: CO. CT. And CO. CT :: CP. CB; wherefore (*ex equo*) CD. CP :: CP. CB; and *for the ſame Reaſon*, CD. CP :: CO. CT; *in like manner*, CP. CB :: CT. CH. Therefore the Lines DO, PT, BH will be parallel to one another, and the Triangles PTB and PTH, which have a common Baſe upon one of the Parallels, and their Altitudes terminated by another, will be equal to one another; from which ſubduct the common Triangle PAT, and the Remainders PAB and TAH will be equal. *W. W.D.*

COROLLARY.

It is alſo evident, upon the ſame Account, that the Triangles PDT, and POT are equal to one another; and if to theſe be added the equal Triangles PTB, PTH, the Quadrilateral Figure POTB will be equal to the Quadrilateral Figure PDTH; or rather, by adding the ſame equal Triangles to the ſame Triangle POT, or PDT, the Quadrilateral Figure POTB will be equal to the Triangle OPH, or the Quadrilateral Figure PDTH equal to the Triangle DTB. It is likewiſe apparent that the Triangle CTB is equal to the Triangle CPH.

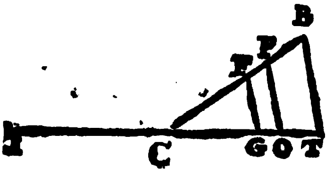
COROLLARY II.

HAVING drawn the Tangent I*b*; it is evident that the Triangles P*ab*, I*a*H are equal; for I*b* is parallel to BT; wherefore the Triangles CI*b*, CTB are ſimilar, and (the Side CI in one being equal to the homologous Side CT in the other) are equal too: Therefore the Triangle CI*b* will be equal to the Triangle CPH, which is equal to the Triangle CTB; and if to theſe equal Triangles be added the common Quadrilateral Figure ICP*a*, the Triangles P*ab*, I*a*H will be equal. But it is further evident, that what hath been demonſtrated in the Triangles comprehended between the Lines CT and CP, may after the ſame manner be demonſtrated of thoſe comprehended between CI and CR; for the Tangent in the Point R muſt be parallel to PH touching the *Ellipſis* in P; for the Triangle CPH is equal to the Triangle CTB, which is equal to the Triangle CI*b*. And ſuppoſing that the Tangent in R meet the *Axis* in *b* (which Point is without the Figure) it will be eaſy to demonſtrate, after the ſame manner as above, that the Triangle CI*b* is equal to the Triangle CR*b*; wherefore (*ex equo*) the Triangle CR*b* is equal to the Triangle PCH; but in theſe equal Triangles, the Angle RCI is equal to the Angle PCH, and CR, one of the Sides which comprehends the equal Angle, is equal to CP of the other (*by Prop.* vi.) wherefore the Triangles are ſimilar, but ſubcontrarily poſited; R*b* therefore is parallel to PH, its homologous Side.

LEMMA.

Let there be a Triangle, as CTB, *whoſe Side* CT *being prolonged to* I, *let* CI *be equal to* CT; *from the Points* O, G, *taken at Pleaſure in the Side* CT, *drawing* OP, GF *parallel to* TB; *I ſay, The Rectangle* IOT, *Is to the Rectangle* IGT :: *As the Quadrilateral Figure* OTBP, *To the Quadrilateral Figure* GTBF.

CT*q*. CO*q* :: Triangle CTB. Triangle COP, and, *by Diviſion*, CT*q* —

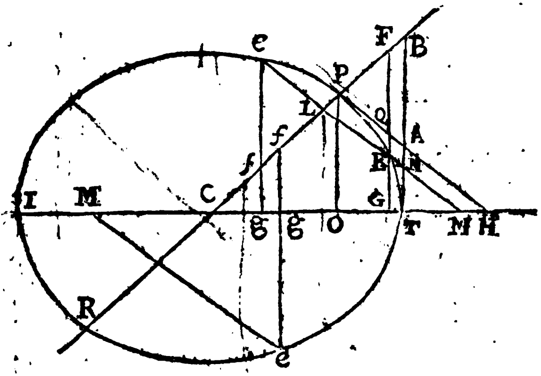


CO*q*. CT*q* :: Triangle CTB — Triangle COP. Triangle CTB, *after the ſame manner* CT*q*. CG*q* :: Triangle CTB. Triangle CGF, and, *by Diviſion*, CT*q*. CT*q* — CG*q* :: Triangle CTB. Triangle CTB — CGF; wherefore, *ex equo*, CT*q* — CO*q*. CT*q* — CG*q* :: Triangle CTB — Triangle COP (which is equal to the Quadrilateral Figure OTBP) Triangle CTB — Triangle CGF (which is the Quadrilateral Figure GTBF) But CT*q* — CO*q* is equal to the Rectangle IOT, and likewiſe CT*q* — CG*q* is equal to the Rectangle IGT, ſeeing IT is divided into equal Parts at the Point C; wherefore the Rectangle IOT. Rectangle IGT :: Quadrilateral Figure OTBP. Quadrilateral Figure GTBF. *Which was to be proved.*

PROP. XIV.

If from any Point E *in the* Ellipſis, *be drawn* LEM *parallel to the Tangent* PH, *and* FEG *parallel to the Tangent* BT; *I ſay, the Triangle* EGM *is equal to the Quadrilateral Figure* GTBF.

By *reaſon of the parallel Lines*, the Triangle POH. Triangle EGM :: PO*q*. EG*q*; but it is evident (*by the third Propoſition*) that PO*q*. EG*q* :: Rectangle IOT. Rectangle IGT; and becauſe IT is divided into two equal Parts in the Point C



(*by the Lemma*) the Rectangle IOT. Rectangle IGT :: Quadrilateral Figure OTBP. Quadrilateral Figure GTBF, and, *ex æquo*, the Triangle POH. Triangle EGM :: Quadrilateral Figure OTBP. Quadrilateral Figure GTBF : But (*by Cor.* i. *Prop. preced.*) the Triangles POH is equal to the Quadrilateral Figure OTBP : Therefore the Triangle EGM is equal to the Quadrilateral Figure GTBF. *W. W. D.*

If the Point E fall in *e* (*See Fig. P.*59) it may be demonſtrated, that the Triangle *eg*M ſhall be equal to the Quadrilateral Figure *g*TB*f*; for *eg* is ſuppoſed to be an Ordinate to the *Axis*. And if the Point *g* cut the *Axis* ſomewhere in CI on the other Hand of C (*See Fig. in Pag.* 62) it may be demonſtrated after the ſame manner, that the Triangle *eg*M is equal to the Quadrilateral Figure *g*I*bf*; for the ſame Demonſtration will ſerve for all, only by conſidering what was ſaid in the ſecond *Corollary* of the *precedent Propoſition.*

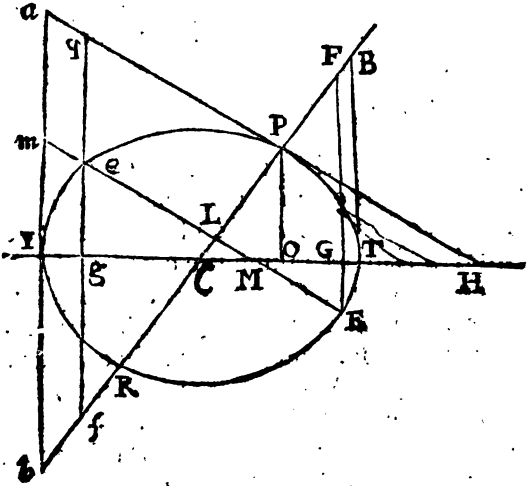
* * *

PROP. XV.

Retaining the firſt Figure in the laſt Propoſition, *the Triangle* ELF, or *e*L*f*, *is equal to the Quadrilateral Figure* LPHM.

1. The Triangle CTB (*by Cor.* i. *Prop.* xiii.) is equal to the Triangle CPH, and (*See Fig. in Pag.* 59) from theſe two equal Triangles taking the common Quadrilateral Figure CGEL, and taking farther from the firſt, the Quadrilateral Figure GTBF; and from the ſecond, the Triangle EGM equal to that Quadrilateral Figure (*by the precedent Propoſition*) there remains the Triangle ELF equal to the Quadrilateral Figure LPMH. *W.W.D.*

2. **If** the Point E be on the other Side of T with regard to P, and the Point L



fall always upon CP; from the equal Triangles CTB, CPH ſubducting the common Triangle CML, and from the firſt Remainder, taking the Quadrilateral Figure GTBF, and adding the Triangle EGM, which is equal to it, we ſhall have the Triangle ELF equal to the Quadrilateral Figure LPHM.

3. But if the Point E be in *e* on the other Side of P with regard to T (*Fig. Prop.* xiv.) and the Ordinate *eg* fall upon

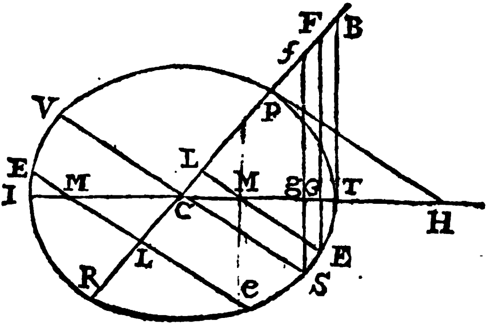
CT

GT the Triangle *eg* M (*by Prop.* xiv.) is equal to the Quadrilateral Figure g T B*f*; and if from theſe Equals, the common Figure *fg* G E L be taken, and farther the Triangle E G M be taken from the firſt, and the Quadrilateral Figure G T B F equal to the Triangle E G M be taken from the ſecond, there remains the Triangle *ef* L = E F L; but the Triangle E F L, from what was demonſtrated above, is equal to the Quadrilateral Figure L P H M; wherefore the Triangle *ef* L is equal to L P H M.

But if in the precedent Caſe the Point M fall upon the *Axis* between the Centre and the Vertex T, the Triangle (*See Fig. P.* 65) *eg* M being equal to the Quadrilateral Figure *g* T B*f*; if the common Quadrilateral Figure *f* g M L be taken from theſe Equals, there will remain the Triangle *e* L *f* equal to the Quadrilateral Figure L M T B; from which if G T B F be taken, and the Triangle E G M, which is equal to it, be added, the Triangle E L F will be equal to the Quadrilateral Figure L M T B; and conſequently the Triangle *e* L *f*, which is equal to the Triangle E L F, is equal to the Quadrilateral Figure L P H M.

In the laſt Place, if the Point g fall upon C I (ſee *Fig. Pag.* 62) the Triangle *a* P *b* is equal to the Triangle *a* I H (*by Cor.* ii. *Prop.* xiii.) and if from theſe two equal Triangles be taken the common Figure *a* P L *eg* I *a*, and farther from the firſt the Quadrilateral Figure *eg* I *bf*, and from the ſecond the equal Triangle *eg* M (*by Prop.* xiv.) there remains the Triangle *e* L *f* equal to the Quadrilateral Figure L P H M. *W. W. D.*

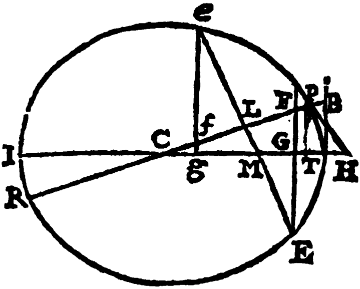
For all the other Caſes, where the Point L falls upon C R, the Demonſtration will be the ſame, as any one will eaſily perceive, that conſiders what we have ſaid in *Prop.* xiii. *Cor.* ii. and if the Point L fall in C, the Point E will be in S or in V (the Line V C S being parallel to P H) and then the Triangle C *f* S will be equal to the Triangle C P H; which I think is more than ſufficient for the Demonſtration of this Propoſition.



PROP. XVI.

In an Ellipſis, *whoſe* Axis *is* IT, *all the Right Lines, as* E e *parallel to a Tangent as* PH, *which meets the Axis in* H, *are cut into two equal Parts in* L, *by the Diameter* RCL *drawn through the Point of* Contact.

HAVING placed all things as in the former *Propoſition*, it is evident, *by the ſame Propoſition*, that the Triangles ELF *e*L*f* are equal to one another; for each of them is equal to the Quadrilateral Figure LPHM, and they are alſo ſimilar, *by reaſon of the Parallels that compoſe them*; wherefore EL = *e*L. *W. W. D.*



PROP.

PROP. XVII.

The ſame Things being ſtill ſuppoſed ; If the Diameter VCS *be drawn parallel to the Tangent* PH; *I ſay, As* VS *Square Is to* RP *Square* :: *So is* EL *Square To the Rectangle* RLP.

THE Triangle CS*f* is equal, *by Prop.* xiv. (*See Fig. in Pag.* 64.) to the Triangle CPH; and the Triangle LEF is equal to the Quadrilateral Figure LPHM; and the Triangles CS*f*, LEF are ſimilar; wherefore they are to one another CS*q*. LE*q*; therefore CS*q*. LE*q* :: Triangle CPH. Quadrilateral Figure LPHM; but becauſe RP is biſſected in C, the Triangle CPH. Quadrilateral Figure LPHM :: CP*q*. Rectangle RLP (*by Lemma Prop.* xiv.) wherefore CS*q*. LE*q* :: CP*q*. Rectangle RLP; or rather CS*q*. CP*q*, or their Quadruples VS*q*. RP*q* :: LE*q*. Rectangle RLP.

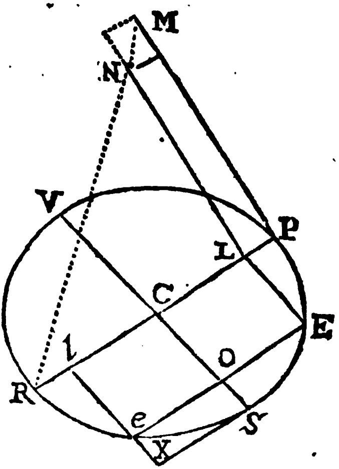
IF the Point E be in *e*, and L fall upon CR, prolong *f* L to L; it may be demonſtrated after the ſame manner that LE*q*. Rectangle RLP :: as VS*q*. RP; but (*by the precedent Prop.*) *e*L = EL; *therefore the Propoſition is true.*

PROP. XVIII.

Still ſuppoſing the ſame Things; If the Right Line E *e be drawn parallel to the Diameter* R P; *I ſay, that* E *e is biſſected in* O *by the Diameter* V S; *and that the Square of* R P *Is to the Square of* V S, *As the Square of* E O *Is to the Rectangle* V O S.

HAVING drawn EL and *e l* parallel to V S (*by the foregoing Propoſition*) E L*q*. Rectangle R L P :: *e l q*. Rectangle R *l* P; but E L = *e l*; wherefore the Rectangle R L P equal Rectangle R *l* P, and conſequently the Lines C L, C *l* are equal, as alſo E O, *e* O, which are equal to them, *which was the firſt Thing propoſed.*

SECONDLY, By the preceding *Propoſition*, C P*q*. C S*q* :: C P*q* — C L*q*. = Rectangle



Rectangle RLP. LE*q*; *by Division* CP*q*, CP*q* — Rectangle = RLP = CL*q*, or EO*q* :: CS*q*. CS*q* — LE*q*, or CO*q*. (which is equal to the Rectangle VOS) therefore CP*q*. CS*q* :: or their Quadruple RP*q*. VS*q* :: OE*q*. Rectangle VOS. *W. W. D.*

COROLLARY.

It is evident that the Line SX parallel to OE, and drawn through S, the Extremity of the Vertex of the Diameter VS touches

Ellipſis in the Point S, which is the Converſe of the preceding Propoſitions.

DEFINITIONS.

I. THE Diameters RP, VS, whoſe Properties are demonſtrated in the two foregoing Propoſitions, are call'd *Conjugates* to one another.

II. THE Line EL parallel to one of the *Conjugate Diameters*, as VS, is called an *Ordinate* to the other *Conjugate Diameter* RP; and reciprocally EO is an *Ordinate* to VS.

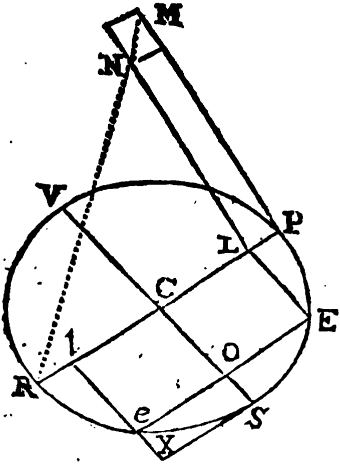
III. IF you make as RP. VS :: VS. PM; this third Proportional PM is called the *Parameter* of the *Diameter* RP; and after the ſame manner for the *Diameter* VS; from whence it is apparent, that the two *Conjugate Diameters* are mean Proportionals between their *Parameters.*

IV. THE Rectangle contain'd under a Diameter RP and its Parameter PM, as RM, is called the *Figure* of the Diameter RP.

PROP. XIX.

The Square of EL, *an* Ordinate *to the* Diameter RP, *is equal to the Rectangle* PN, *apply'd to the* Parameter *of this* Diameter, *which hath* PL *for its Altitude, and is leſs than the Rectangle* LM *by the Rectangle* NM, *which is ſimilar to, and poſited ſimilar to the Figure* RM.

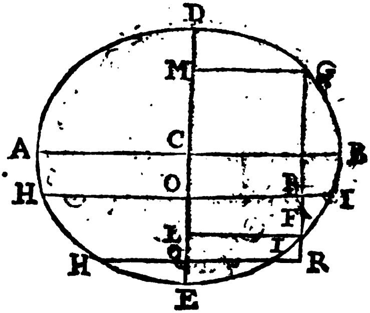
By either of the *two foregoing Propoſitions*, RP*q*. VS*q* :: Rectangle RLP LE*q*; But RP*q*. VS*q* :: RP. PM, and



and R P. P M :: R L. L N; but Rectangle R L P. Rectangle N L P :: R L. N L; wherefore Rectangle R L P. L E *q* :: Rectangle R L P. Rectangle N L P; the Square of the Ordinate E L therefore is equal to the Rectangle N L P. *W. W. D.*

PROP. XX.

In the Ellipſis A D B E, *if two Right Lines* H I, F G *be drawn parallel to two Conjugate Diameters* A B, D E, *meeting one another in* R, *a Point without the* Ellipſis; *I ſay, the Rectangle* H R I. *Rectangle* F R G :: A B*q*. D E*q*.



Having drawn F L, G M *Ordinates* to the *Diameter* E D, they will be parallel to A B; (*by Prop.* xvi, *or* xvii.) and L F *q*. O I *q* :: Rectangle E L D. Rectangle

EOD (by the ſame *Propoſition*) and if the Point R be without the *Ellipſis*; *by Diviſion*, LF*q* — OI*q* = Rectangle HRI. OI*q* :: Rectangle ELD — Rectangle EOD = Rectangle LOM or FRG. EOD. But if the Point R be in the *Ellipſis*; *by Diviſion*, OI*q* — LF*q* or OR*q* = Rectangle HRI. OI*q* :: Rectangle EOD — Rectangle ELD = Rectangle LOM or FRG. Rectangle EOD; But OI*q*. (*by Prop.* xvi, xvii.) Rectangle EOD :: AB*q*. ED*q*; wherefore the Rectangle HRI. Rectangle FRG :: AB*q*. ED*q*. *W. W. D.*

A
TREATISE
OF
CONICK SECTIONS.

PART III.

The *HYPERBOLA*.

Of the GENERATION *of the* HYPERBOLA.

If upon a Plane the Right Line IT *be drawn, and biſſected in* C, *and the Points* F *and* D *be taken at equal Diſtances from* C *in the ſame Line produc'd both Ways;*

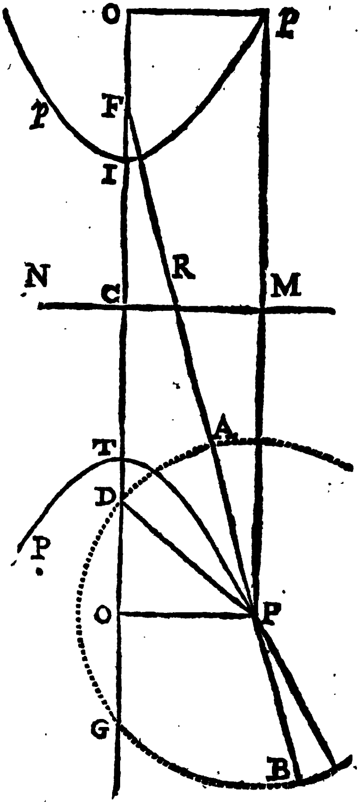
you may find as many Points as you pleaſe, as P *p*, *ſo diſpoſed that the Right Line* P F *ſhall exceed* P D *by the Line* T, *or* *p* D *ſhall exceed* *p* F *by the ſame Line* I T.

TAKE FP as great as you pleaſe, provided it be not leſs than F T, and then the Difference betwixt F P and I T will be always greater, or at leaſt equal to D T; wherefore if the Point F be made the Centre of a Circle, whoſe Radius let be FP, and D the Centre of another Circle, whoſe Radius let be DP, theſe two Circles will neceſſarily interſect one another in two Points, as P, on each Side the Line I T; or at leaſt will touch one another in the Point T, in caſe P and T coincide; for then F T and D T will be the two Radii; and thus you will find different Points as P, if you take the Lines F P different from one another. After the ſame manner may be found as many Points *p* as ſhall be deſired.

COROLLARY.

IT is evident, *by this Generation*, that the Right Line which joyns the two Points P or *p*, the Interſections of the two Circles,

cles, will be perpendicular to F D, and biſſected by it: Wherefore it is evident



that the Points P, p will form a curved Line, which will paſs through T, and

the

the Points P, *p* another, which will paſs through I: Moreover, any one will eaſily ſee, that the Lines P T P, and *p* I *p* will not include Space, neither both together, nor one alone; and that they encreaſe infinitely, and run off continually from the Line I T, and are continued from the Point C, one on one Side, and the other on the other.

DEFINITIONS.

I. THE Curved Lines P T P, *p* I *p*, are each of them ſeparately called an *Hyperbola*, and conjoyntly *Oppoſite Hyperbola's.*

II. THE Point C is called the Centre of the *Hyperbola*, or of the *Oppoſite Hyperbola's.*

III. THE Right Line I T is called the *Determinate Axis.*

IV. THE Right Line N C M, which paſſes through the Centre C, and is perpendicular to I T, is called the *Indeterminate Axis.*

V. THE Points F and D, the *Foci.*

VI. THE Right Lines, as P O, drawn from the Points of one *Hyperbola*, or the

Oppoſite *Hyperbola's* perpendicular to one of their *Axes*, are called *Ordinates* to that *Axis*.

VII. All the Right Lines which paſs through the Centre C, are called *Diameters*, thoſe which meet the Oppoſite Hyperbola's are *Determined*, and thoſe which do not, are *Indetermined*.

VIII. A Right Line which meets the Hyperbola but in one Point, and does not paſs within it, is called a *Tangent* to it in that Point.

PROP. I.

The Hyperbola *being formed, I ſay, that the Square of* PO, *an* Ordinate *to the* determinate Axis IT, *is to the Rectangle* IOT, *as the Rectangle* IFT, *or* TDI, *which is equal to it, to the Rectangle* ICT, *which is the Square of* CT.

Draw the Line FP, and prolong it to B, ſo that PB may be equal to PD; on P as a Centre, and with the Radius PD deſcribe the Circle BGD, cutting FP in A, and IT in D and G: FA will be equal to IT, *by the Generation*; which let be divided into two equal Parts in R. By reaſon of the Circle $ADGB$, the Rectangle $FA \times FB =$ the Rectangle $FD \times FG$. Wherefore $FA . FD :: FG . FB$, and conſequently their Halves FR or $CT . CD :: CO . RP$; and, *by Compoſition*, $CT . CT + CD :: CO . CO + RP$; and *alternately* $CT . CO :: CT + CD . CO + RP$; and farther, *by Compoſition*, $CT . CT + CO = IO :: CT + CD = ID . CT + CD = ID + CO + RP$.: But $CT + CD + CO + RP$, or their Equals FR, FC, CO, RP being added together, are $= FP + FO$; wherefore $CT . LO :: ID . FP + FO$.

In like manner, taking the Proportion above, $CT . CD :: CO . RP$. *by Diviſion*, $CT . CD - CT :: CO . RP - CO$, and *alternately* $CT . CO :: CD - CT . RP - CO$; but *farther*, *by Diviſion*, $CT . CO - CT (= OT) :: CD - CT (= DT.) RP - CO - CD + CT$: But $RP - CO - CD + CT$, or their Equals $RP - CO - FC + FR$, are equal to the Difference of FP and FO; wherefore $CT . OT :: DT . FP - FO$.

In

If the laſt Proportions, in the two preceding Sections, be multiply'd into one another, Antecedents into Antecedents, and Conſequents into Conſequents, we ſhall have $CTq.$ Rectangle $IOT :: $ Rectangle $IDT.$ Rectangle $FP + FO \times FP - FO = FPq - FOq = POq$ (*by the Lemma, Prop.* i. *Ellipſis*) W. W. D.

If the Circle BDA touch IT in D, *i. e.* if the Point O fall upon the Point D, which is the ſame thing, the Demonſtration will be the ſame, but ſtill more ſimple; for by that means, more different Quantities would become equal.

COROLLARY.

It is evident, that the Squares of the *Ordinates* to the *Axis*, are to one another as the Rectangles contain'd under the Parts of that *Axis*, comprehended between its Extremity and the Points where it meets thoſe *Ordinates*.

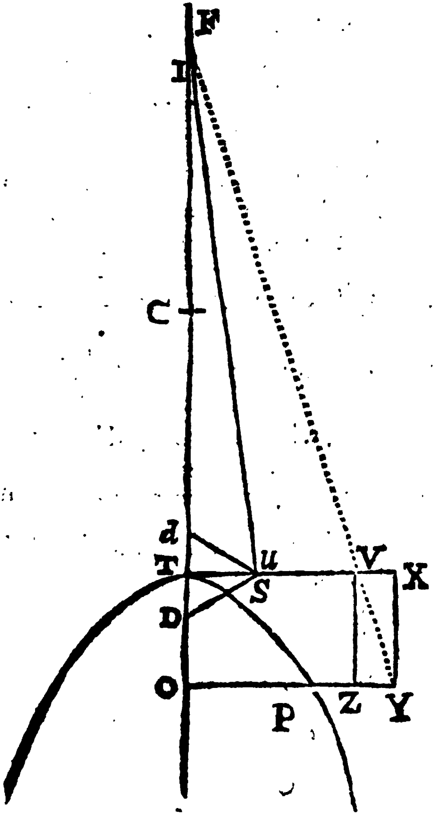
PROP. II.

The Right Line P*p* *drawn between the oppoſite* Hyperbola's, *parallel to the* determinate Axis I T, *meets the* indeterminate Axis N M *in a Point, as* P M; *which Point* M *divides* P *p* *into two equal Parts.*

THIS Propoſition is ſufficiently evident *from the Conſtruction* (*See Fig. Pag.* 75) for if D be made the Centre of a Circle, and F P its Radius, F the Centre of another Circle, and D P its Radius, the two Circles will interſect one another at the Point *p*; ſo that *p o* being drawn at Right Angles to the *Axis* I T, will be equal to P O, and likewiſe *p* M will be = P M.

DEFINITION.

IF you make as C T *q*. Rectangle I D T :: C T. T *u*; the Line T V double of



of T*u* is called the *Parameter* of the *determinate Axis*.

And the Rectangle IV, made by the *Axis* IT and its *Parameter*, is called the Figure of that *Axis*.

COROLLARY.

It is evident, that the Figure IV is equal to four times the Rectangle IDT; for putting CT, or CI $= a$, and DT $= x$; then, by the Definition of the *Parameter*, $a^2 . 2ax + xx :: a . \frac{2a^2x + axx}{a^2}$; which being doubled, gives $\frac{4a^2x + 2axx}{a^2} =$ *Parameter*; which multiply'd by $2a =$ IT, gives $\frac{8a^3x + 4a^2xx}{a^2}$ which is manifeſtly quadruple the Rectangle IDT $= 2ax + xx$.

PROP.

PROP. III.

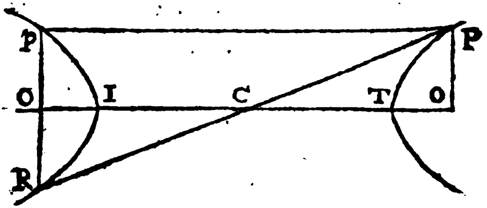
The Square of PO, *an Ordinate to the* determinate Axis IT, *is equal to the Rectangle* OTXY, *apply'd to the Parameter* TV; *the Altitude of which Rectangle is* TO, *the Part contained between the Extremity* T *and the Foot of the Ordinate* O; *which Rectangle exceeds the Rectangle* TOZV *by the Rectangle* VZYX, *which is ſimilar and ſimilarly poſited to the Figure. See the preceding Figure.*

Retaining the ſame Figure CT*q*. Rectangle IDT :: Rectangle IOT. PO*q*; but by the preceding *Cor. Prop.* ii. CT*q* IDT :: IT*q*. ITV :: IT. TV; wherefore, *ex æquo*, IT. TV :: Rectangle IOTPO*q*; but IOT. YOT :: IO. YO :: IT. TV; therefore, *ex æquo*, Rectangle IOT. PO*q* :: Rectangle IOT. Rectangle YOT; and conſequently YOT = PO*q*. *W. W. D.*

PROP. IV.

If from any Point *of one of the oppoſite* Hyperbola's, *be drawn the Diameter* PC, *and continued beyond* C, *it will meet the other of the oppoſite* Hyperbola's, *and likewiſe be biſſected in* C.

DRAW PO an *Ordinate* to the *Axis*



IT, and make C*o* = CO; and draw likewiſe the *Ordinate po*R on each Side of the *Axis*; RO = PO (*by Prop.* ii.) wherefore RP will paſs through the Centre, and will but make one Right Line with PC, which was firſt drawn, and will alſo be biſſected in C. *W. W. D.*

PROP. V.

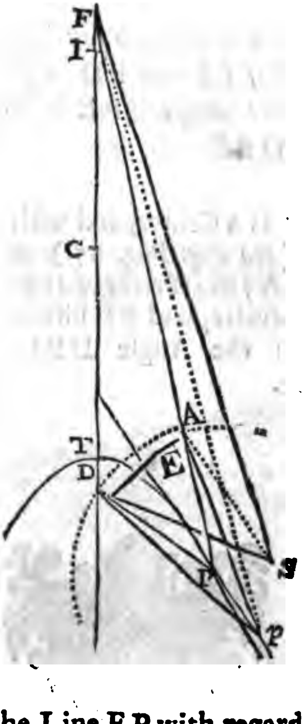
A Perpendicular to the Axis, *as* TX, *meeting the* Hyperbola *in the* Vertex T, *touches the* Hyperbola *in that Point.*

Suppose XT do not touch the *Hyperbola*, but meet it in ſome other Point, as S, which is different from T (*See Fig. Pag:* 82.) take $Td = TD$, and then will Fd be $= IT$, and $Sd = SD$; but (*by the Generation of the* Hyperbola) $IT + SD$, or their Equals $Fd + Sd = FS$, *i. e.* one Side of the Triangle FSd is equal to the Sum of the other two Sides Fd, dS, *which is abſurd*; therefore the Perpendicular XT does not meet the *Hyperbola* except in the Point T: I ſay alſo, that it can never poſſibly paſs within the *Hyperbola*, though it be produced; for it is evident (*from the Generation*) that the *Hyperbola* runs off more and more from the indeterminate *Axis*, which is parallel to XT; therefore XT touches the *Hyperbola* in T; *which was to be ſhewn.*

PROP. I.

Things being put (as in the firſt Propoſition) *and having drawn* D A, *and biſſected it in* E; *I ſay, the right Line* P E *touches the* Hyperbola *at the Point* P.

SUPPOSE it otherwiſe, and that P E meet the *Hyperbola* in any other Point, as *p*, then (*by the Generation of the* Hyperbola) the Difference of the Lines F *p*, *p* D equal to the Difference of the Lines F P, P D is equal F A; *by the Hypotheſis* P E is perpendicular to A D, and is biſſected in E; wherefore *p* A = *p* D; therefore the Difference of the Sides F *p* and *p* A is equal to the other Side F A, *which is abſurd.* But if the right Line E P produced beyond P, fall within the *Hyperbola*, it is manifeſt, that the *Hyperbola* will paſs on the other Side of E P, with Regard to the *Axis* I T; therefore having drawn from any of its Points, as S, the Lines S F and S D to the *Foci* (*by the Generation*) the Line I T (or F A, which is equal to it) + S D is = S F; but the Point S being without



without the Line E P with regard to the Point D, the Line S A muſt be leſs than S D; wherefore S A + F A will be leſs than S F, *which is abſurd*, for two Sides of any Triangle are always greater than the third; *therefore the* Prop. *is true.*

PROP. VII.

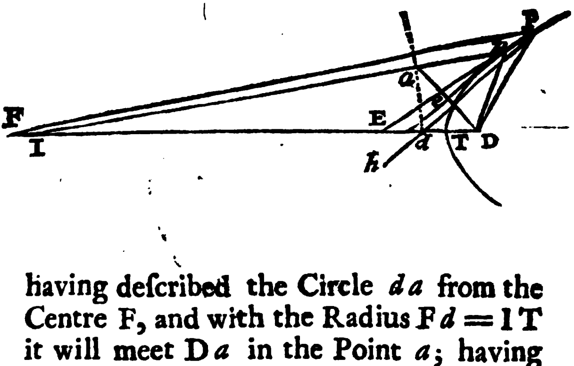
If the Line PE *touch an* Hyperbola *in* P, *and if* PF *and* PD *be drawn to the* Foci, *the Angle* FPE *is equal to the Angle* DPE.

On P as a Centre, and with the Radius PD (*See Fig. Pag.* 87.) deſcribe the Circle DA; the Triangle DPA (*by Prop.* vi.) is Iſoſcelar, and PE biſſects the Baſe; wherefore the Angle DPE = FPE. *W. W. D.*

PROP. VIII.

The ſame Things being put (as in Prop. vi.) *I ſay there can be but one Line, as* P E, *which ſhall touch the* Hyperbola *in the ſame Point p.*

LET P *b* alſo touch the *Hyperbola* in P, *if it be poſſible.* From the *Focus* D having drawn *D a* perpendicular to P *b*, and

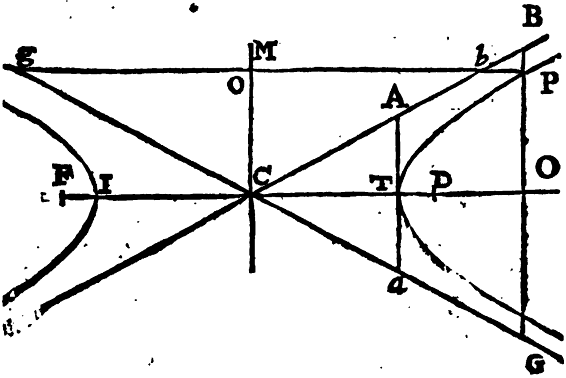


having deſcribed the Circle *d a* from the Centre F, and with the Radius F *d* = I T it will meet D *a* in the Point *a*; having biſſected D *a* in *e*, and having drawn F *a* produced until it meet the *Hyperbola* in *p*; and having join'd *p* D and *p e*, *a p* and *p* D will be equal (*by the Generation*) and *p e* drawn within the Iſoſceles Triangle *a p* D biſſecting the Baſe D *a* will be perpendicular to D *a*; wherefore it will be parallel to P *b*; but it touches the *Hyperbola* in *p* (*by Prop.* vi.) therefore *p e* will meet a Line touching the *Hyperbola* in the Point T, in ſome Point without the *Hyperbola*, ſeeing they are both Tangents; wherefore the Parallel P *b* paſſes within the *Hyperbola*, for the *Hyperbola* goes from P to *p*, and the *Ordinate* to the Point P, meets P *b* within the *Hyperbola*, *which is abſurd*; for P *b* was ſuppoſed a Tangent: Therefore there can be drawn but one Line, as *p*E, which will touch the *Hyperbola* in the ſame Point *p*. *W. W. D.*

Definition

Definition of the Aſſymptotes.

IF the Right Line *a*TA be drawn perpendicular to the *Axis* IT, at one of its



Extremities, as T, and TA*q* be made equal to the Rectangle IDT; having taken T*a* = TA; thro' the Centre C and the Points A, *a* draw the Lines AC, *a*C indetermined both ways from the Centre; theſe Lines are called *Aſſymptotes* to the *Hyperbola*, or the oppoſite *Hyperbola's*.

COROLLARY.

IT is evident that the *Aſſymptotes* to one *Hyperbola* are alſo *Aſſymptotes* to the other that is oppoſite.

PROP. IX.

If the Ordinate *of an* Hyperbola *be produced until it meet the* Aſſymptotes *in* B *and* G; *the Rectangle* GPB *is equal to* TAq. (See Fig. Pag. 91.)

THE Lines TA and OB are parallel; wherefore CTq. TAq :: COq. OBq, and CTq. TAq :: COq — CTq = Rectangle IOT. OPq. and, *ex æquo*, COq, OBq :: COq. — CTq. OPq, and *alternately* COq. COq — CTq :: OBq. OPq, and, *by Diviſion*, COq. COq — COq + CTq = CTq :: OBq. OBq — OPq, and *alternately* COq. OBq :: CTq. OBq — OPq = Rectangle GPB : But *in the firſt Propoſition* COq. OBq :: CTq. TAq; wherefore the Rectangle GBP = TAq. *W. W. D.*

PROP. X.

If from ſome Point, as P, *in a* Hyperbola *be drawn the right Line* Pg *parallel to the* Axis IT, *meeting the two* Aſſymptotes *in b and* g; *I ſay, the Rectangle* gP*b is equal to* CT*q*.

FROM *the Poſition of the* Aſſymptotes, is biſſected in O by the *Axis* CM; and *from ſimilar Triangles*, OB*q*. (*See Fig. in Pag.* 91.) CO*q* = PO*q* :: CO*q*, or *o*P*q*. *obq*; and *by Diviſion*, OB*q*. OB*q* — OP*q* :: *o*P*q*. *o*P*q* — *obq*; and *alternately*, and *by Inverſion*, OB*q* — OP*q* = Rectangle GPB. *o*P*q* — *obq* = Rectangle *g*P*b* :: OB*q*. *o*P*q* = CO*q*: But OB*q*. CO*q* :: TA*q*. CT*q*; wherefore, *ex æquo*, Rectangle GPB. Rectangle *g*P*b* :: TA*q*. CT*q*; but Rectangle GPB = TA*q*. (*by the laſt Prop.*) wherefore Rectangle *g*P*b* = CT*q*. *W. W. D.*

PROP. XI.

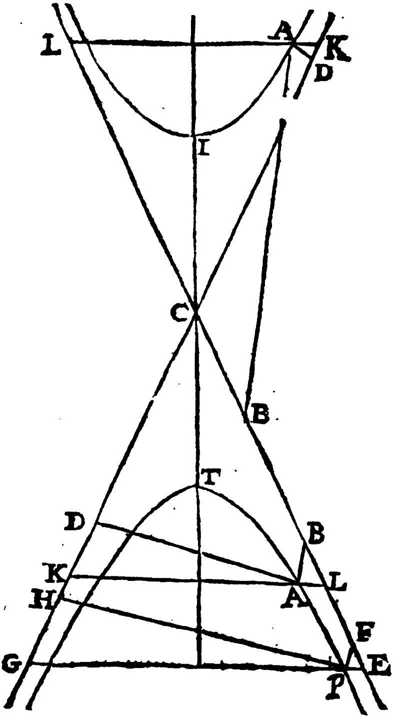
The Hyperbola *and its* Aſſymptote *approach continually to one another the farther both of them are produced, and yet will never meet, tho'* P B, *the Part of the* Ordinate *contain'd between the* Hyperbola *and its* Aſſymptote, *may be found ſuch, that it ſhall be leſs than any Line given.*

THE Rectangle G P B is equal to T A*q*. (by *Prop.* ix.) (*See Fig in Pag.* 91.) but G B encreaſes the farther it is diſtant from C; wherefore P B continually diminiſheth; but it can never become equal to nothing, for then the Rectangle G P B would be equal to nothing, which wou'd contradict the *forementioned Prop.* wherefore the *Hyperbola* and its *Aſſymptote* can never meet: Then if a Rectangle be made, one of whoſe Sides ſhall be leſs than the given Line, which Rectangle ſhall be equal to T A *q*; the Sum of the two Sides of this Rectangle, as G B, being applied within the Angle made by the *Aſſymptotes* perpendicularly to the *Axis* I T, P the Point that divides the Sides of that Rectangle will be within the *Hyperbola* (by *Prop.* ix.) and conſequently PB is leſs than the given Line. *W. W. D.*

PROP. XII.

If two Points, as P and A, *be taken in a* Hyperbola, *or one Point only in each of the oppoſite* Hyperbola's, *and through theſe Points be drawn two right and parallel Lines, as* PH *and* AD *terminated by one of the* Aſſymptotes, *as* CD; *in like Manner, if thro' the ſame Points* P, A *be drawn two other Lines* PF, AB *parallel to one another, and terminated at the other* Aſſymptote CB; *I ſay, the Rectangle* PH × PF = AD × AB.

THROUGH the Points P, and A, having drawn the Perpendiculars to the *Axis* EPG, and LAK terminated by the *Aſſymptotes* in EG, LK; the Triangles EPF



EPF and LAB, PGH and KHD *are ſimilar*, by Reaſon the Lines that compoſe them are parallel; wherefore EP. PF : : LA. AB, and PG. PH : : AK. AD; wherefore multiplying theſe two Proportions into one another, Antecedents into

Antecedents, and Conſequents into Conſequents; there will ariſe this Proportion EP × PG. PF × PH :: LA × AK. AB × AD; but the Rectangle EPG equal Rectangle LAK; for (by *Prop.* ix.) they are equal Squares of one and the ſame right Line; wherefore the Rectangle PF × PH is equal to the Rectangle AB × AD. *W. W. D.*

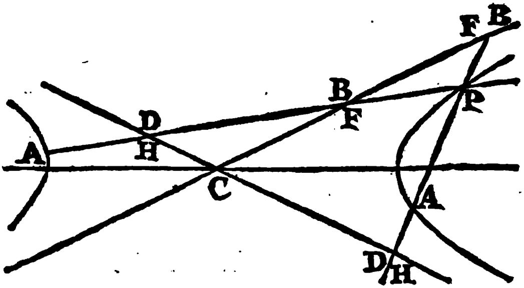
COROLLARY.

It is evident that what we have demonſtrated above for two Points only, may be demonſtrated in like manner for any Number of Points whatſoever.

PROP. XIII.

Draw a right Line at Pleaſure, as PA, *meeting one* Hyperbola, *or the two oppoſite* Hyperbola's *in the Points* P *and* A, *and the* Aſſymptotes F *and* H, *and then the Parts of this Line* PF, AH, *comprehended between the* Hyperbola, *or the* Oppoſite Hyperbola's *and their* Aſſymptotes *are equal.*

By the preceding Prop. the Rectangle



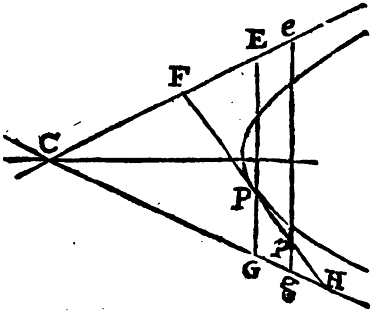
P F x P H is equal to the Rectangle A B x A H; for the Lines P F, A B, as alſo P H and A D are parallel, being join'd in a ſtreight Line: But the Sums or the Differences of H F, D B, the Sides of thoſe equal Rectangles, are alſo equal, being they are the ſame right Lines; wherefore the Sides of theſe Rectangles are alſo equal, *viz.* P F = A D, and P H = A B.

PROP. XIV.

If a right Line, as F H, *touch an* Hyperbola *in* P; *I ſay, that it meets the Aſſymptotes in* F *and* H, *and is biſſected by the Point of Contact* P.

1. Thro' the Point P draw E P G an *Ordinate* to the *Axis*, and if the Tangent in P do

P do not meet the *Aſſymptotes*, it will be



parallel to one of 'em, which let be CG, *if it be poſſible.* (*By Prop.* xi.) one may find a Line leſs than PG, which being parallel to CG, let it be comprehended between the *Hyperbola* and *the Aſſymptote*; this Line produced will neceſſarily meet the Tangent FPH parallel to CG within the *Hyperbola*, *which is abſurd*; for FPH is ſuppoſed to be a Tangent; therefore a Tangent meets both the *Aſſymptotes*, *which is the firſt Part of the Prop.*

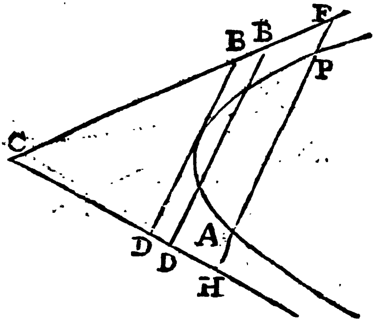
2. If PH be unequal to PF, let it be greater if poſſible; having cut off $Hp = FP$, draw *epg* parallel to EPG; then, becauſe $pF = Hp$, and *ge* parallel to GE, $pg.\ PG :: Hp.\ HP$; and $Hp.\ HP$, or

their Equals $FP . Fp :: PE . pe$, and, *ex æquo*, $pg . PG :: PE . pe$; wherefore $PG \times PE = pg \times pe$; and *by the Converſe* of *the* 9th. *Prop.* the Point *p* will be one of the Points of the *Hyperbola*; therefore FPH will meet the *Hyperbola* in two Points P, *p contrary to the Hypotheſis*, for it is ſuppoſed to be a Tangent. *Wherefore the Prop. is true.*

PROP. XV.

Thro' the Points P, A *of an* Hyperbola, *draw the right Lines* FH, BD *parallel to one another, and meeting the* Aſſymptotes *each of them in the Points* F, H, B, D; *I ſay, the Rectangle* $PF \times PH$ *is equal to the Rectangle* $AB \times AD$ *or* ABq, *if* BD *touch the* Hyperbola *in the Point* A.

This Propoſition is evident from *Prop.* xii *and* xiv; for the Lines PF, PH, and AB,

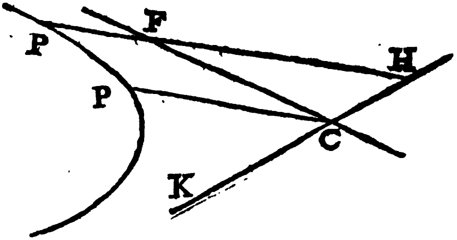


AB, AD are parallel; and if BD be a Tangent in the Point A, it is divided by the Point A into two equal Parts (*by Prop.* xiv.)

PROP. XVI.

If from any Point P *of an* Hyperbola *be drawn* A *right Line parallel to the determinate Diameter, as* CA; *I ſay,* PH *meets the* Aſſymptotes *in* F *and* H, *and the Rectangle* PF × PH = CA*q*.

For CA is within the Angle made by

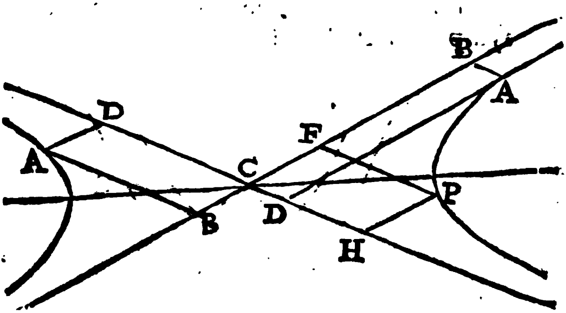


the *Aſſymptotes*; therefore it is manifeſt, that PH which is parallel to CA, muſt meet the *Aſſymptotes* in F and H: But (*by Prop.* xii.) the Lines AC and PF being parallel, and meeting the *Aſſymptote* CF; and AC and PH being alſo parallel, and meeting the other *Aſſymptote* CH, the Rectangle AC × AC = AC*q* is = Rectangle PF × PH. *W. W. D.*

PROP. XVII.

If from the Points P, A *of an* Hyperbola *or* oppoſite Hyperbola's, *be drawn* PF, PH, *and* AB, AD *parallel to its* Aſſymptotes, *the Parallelogram* PFCH *is equal to the Parallelogram* ABCD.

By Prop. xii. the Rectangle PF × PH

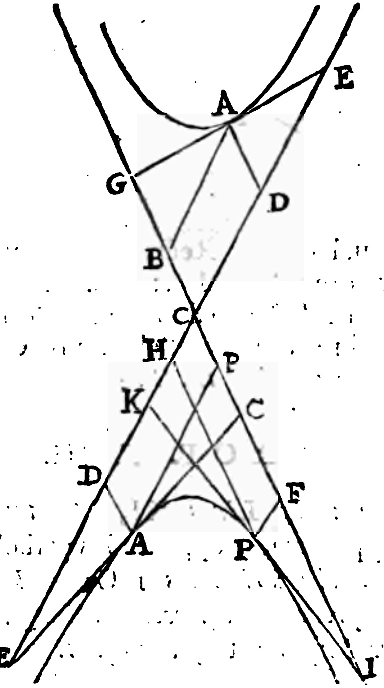


is equal to the Rectangle AB × AD; wherefore PH. AD :: AB. PF; but the Angle FPH is equal to the Angle BAD; wherefore the Parallelograms CP and CA are equal. *W. W. D.*

PROP. XVIII.

If the right Lines KI, EG *touch an* Hyperbola, *or* oppoſite Hyperbola's; *I ſay, the Triangles* KCI *and* EGC, *comprehended between the Tangents and the* Aſſymptotes, *are equal.*

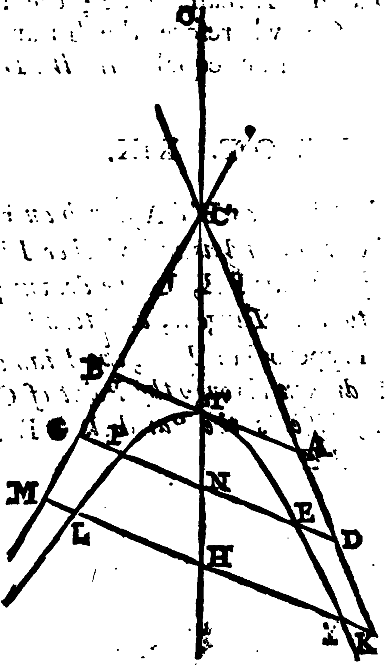
THROUGH the Points of Contact P and A, having drawn the Parallels to the Aſſymptotes AB, AD, and PF, PH (*by the precedent Prop.*) the Parallelograms CA and CP are equal; but becauſe the Tangents are biſſected by the Points of Contact (*by Prop.* xiv.) the Parallelograms CP ſhall



ſhall be $\frac{1}{2}$ the Triangle CIK; and the Parallelogram CA ſhall alſo be $\frac{1}{2}$ the Triangle CEG; wherefore the Triangles CIK and CEG are equal. *W. W. D.*

PROP. XIX.

If a right Line, as BTA, *touch an* Hyperbola *in* T, *and as many other Lines as you pleaſe, as* PE, LI *be drawn parallel to this Tangent, and terminated in the* Hyperbola; *I ſay, the Diameter* OCT *drawn through the Point of Contact* T, *biſſects the Parallels* PE, LI *in* N *and* H.

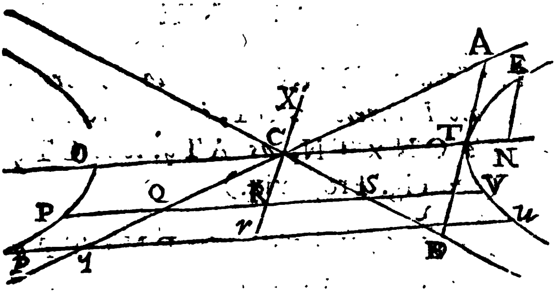


THE Tangent BA is biſſected in T (*by Prop.* xiv.) wherefore PE, LI being produced to the *Aſſymptotes* in G,D,M,K, GD and MK will alſo be biſſected in the Points N and H, by the Diameter CT; but GP = ED and ML = IK (*by Prop.* xiii.) wherefore PN = NE, and LH = HI. *W. W. D.*

PROP.

PROP. XX.

If a right Line, as A B, *touch an* Hyperbola *in the Point* T, *draw the Diameter* XCR *parallel to the Tangent* AB, *and another Diameter, as* OCT, *paſſing thro' the Point of Contact* T; *I ſay, that all theſe Parallels, as* PV, *pu to the Diameter* OT, *and terminated between the oppoſite* Hyperbola's, *are biſſected in* R, *and r by the Diameter* XCR.



Because AB, which is parallel to XC, is biſſected in T, it follows that QS, and *q s* comprehended between the *Aſſymptotes*, and the Parallels to CT, will likewiſe be biſſected in R and *r*; but PQ = SV, and *pq* = *su*; wherefore PR = RV, and *pr* = *ru*. *W. W. D.*

DEFINITIONS.

1. THE Diameters O T, X R whoſe Properties we have demonſtrated in the two preceding Propoſitions, are called *Conjugates* to one another (*See Fig. Prop.* xix, *and* xx.

2. A RIGHT Line, as E N, parallel to one of the Conjugates, as X C, is called an *Ordinate* to the other O T, and reciprocally V R is an *Ordinate* to X C.

PROP. XXI.

The Square of E N, *Ordinate to a determinate Diameter as* O T, *is to the Rectangle* O N × T N, *as* A T*q. is to* C T *q* (See Fig. Prop. xix.)

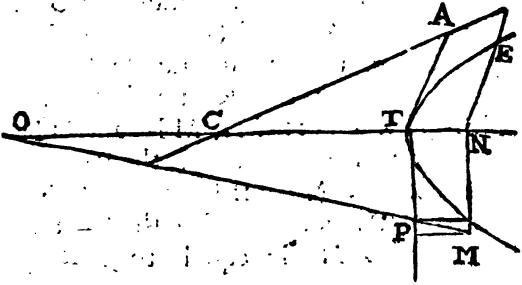
FROM *ſimilar Triangles* D N *q*. T A *q* :: C N *q*. C T *q* ; *and by Diviſion* D N *q* — T A *q* = G E D by *Prop.* xv. (which is = E N *q*) is to T A *q* :: C N *q* — C T *q* (equal to the Rectangle O N × T N) C T *q* ; wherefore E N *q* T A *q* :: O N × T N. C T *q*. *W. W. D.*

COROLLARY.

It is evident, that the Squares of the Ordinates EN, IH to a determinate Diameter OT, are as the Rectangles ON × TN, OH × TH, made under the Parts of that Diameter, which are comprehended between its Extremities O and T, and N and H the Points where the Ordinates meet it.

DEFINITIONS.

1. Let CA be to TA as TA is to



half of TP; the right Line TP is called the *Parameter* of the determinate Diameter OT.

2. The Rectangle OP contained under the determinate Diameter OT and it's *Parameter*, is called the *Figure* of that Diameter OT.

PROP.

PROP. XXII.

The Square of EN, *an Ordinate to a determinate Diameter, as* OT, *is equal to the Rectangle* TM *applied to the* Parameter TP, *whoſe Altitude is* TN *the* Abſciſſa, *which Rectangle exceeds* PN *by the Rectangle* PM, *which is ſimilar and ſimilarly poſited to the Figure* OP. (See Fig. preced. Defin.)

By *the Formation of the* Parameter, OT. TP :: CT*q*. TA*q*. and (*by the precedent Prop.*) CT*q*. TA*q* :: Rectangle ON × TN. NE*q*; but the Rectangle ON × TN. MN × TN :: ON. MN, and ON. MN :: OT. TP, and *ex æqua* the Rectangle ON × TN. NE*q* :: Rectangle ON × TN. MN × TN; wherefore the Rectangle MN × TN is equal to NE*q*. *W. W. D*

COROLLARY.

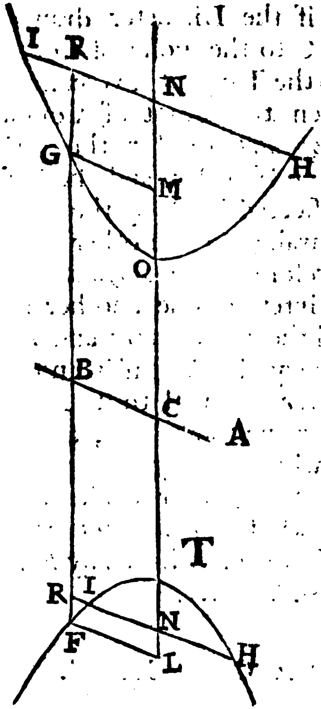
If the *Parameter* TP be equal to the Diameter OT, it follows, from the Formation of the *Parameter*, that TA will be equal to CT; *i.e.* equal ½ the ſame *Parameter*. But it is alſo evident (*by Prop.* xviii.)

xviii.) that the Triangle A B G is rectangled in B, if the Diameter drawn from the Centre C to the Point of Contact A, be equal to the Tangent A G, comprehended between the Point of Contact A, and the *Aſſymptote*; for the Triangle C A G will be Iſoſcelar, and the Baſe C G muſt be biſſected in B, by the Line A B, which is parallel to the other *Aſſymptote* C E; wherefore if the *Aſſymptotes* of an *Hyperbola* interſect one another at right Angles, all their *Diameters* and *Parameters* will be equal: And if in an *Hyperbola* a *Diameter* be equal to its *Parameter*, all the others will be ſo too, and the *Aſſymptotes* will be at right Angles. And moreover, it is farther evident, that an *Hyperbola* whoſe *Aſſymptotes* are not at right Angles, cannot have any *Parameter* equal to its *Diameter*.

PROP. XXIII.

If in the oppoſite Hyperbola's, *two right Lines* H I, F G *parallel to two Conjugate Diameters* A B, O T, *meet in a Point as* R; *I ſay, the Rectangle* F R G. *Rectangle* H R I :: *Diameter* O T. *to its* Parameter.

HAVING



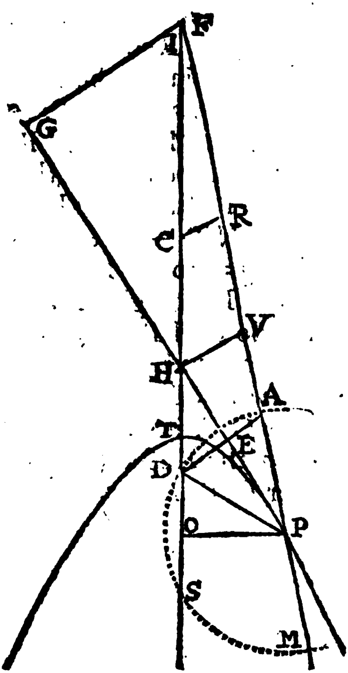
Having drawn FL and GM Ordinates to the Diameter OT, they will be parallel to AB (*by the Definition of conjugate Diameters*) and OT biſſects HI in N (*by Prop.* xix.)

IN like manner G F is biſſected by the Diameter A B; but C O = C T, and M L = G F; wherefore T L = O M (*by Prop.* xxi.) L F *q*. N I *q* :: Rectangle O L T. Rectangle O N T; and, *by Diviſion*, if the Point R be without the *oppoſite Hyperbola's* L F *q* — N I *q* (= Rectangle H R I.) is to N I *q* :: Rectangle O L T. — Rectangle O N T (= M N L or F R G; for M O and T L are equal) O N T; therefore the Rectangle H R I. N I *q* :: Rectangle F R G. Rectangle O N T.

BUT if the Point R be within one of the oppoſite *Hyperbola's*, *by Diviſion*, N I *q* — L F *q*, or G M *q*, or N R *q* (L F, G M, and N R being all three equal) equal to the Rectangle H R I : N I *q* :: Rectangle O N T — Rectangle O L T (O L T being = to the Rectangle O M T which is = Rectangle F R G) is to the Rectangle O N T. *But in theſe two Caſes* (*by Prop.* xxi.) and the Definition of the *Parameter*, Rectangle O N T. N I *q* :: Diameter O T. it's *Parameter*; wherefore *ex æquo* the Rectangle F R G. H R I :: Diameter O T. its *Parameter W. W. D.*

PROP. XXIV.

In an Hyperbola, *as* PT, *whoſe* Axis *is* IT, *and its Centre* C, *let* H *be the Point where a Tangent, as* PH, *meets the* Axis, *and let* PO *be an Ordinate to the* Axis *paſſing thro' the Point of Contact* P; *I ſay,* CH. CT :: CT. CO.



LET the Points F, D be the Foci, and from P as a Centre; and with the Radius P D, having deſcribed the Circle M S D A, and drawn the Line D A: the Tangent PH will biſſect D A in E (*by Prop.* vi.) From the Points F, C, H, having drawn F G, C R and H V parallel to D A, F A will be biſſected in R, becauſe F D is biſſected in C.

FROM *ſimilar Triangles*, F P. P A : : F G. A E, or D E equal to it, and F G. D E : : F H. D H, and F H. D H : : F V. V A; conſequently *ex æquo* F P. P A : : F V. V A; *by Diviſion*, F P — P A = F A. P A : : F V — V A = 2 R V. V A; but F A. 2 R V : : as *their Halves*, viz. R A R V. and *ex æquo* R A. P A : : R V. V A; *by Compoſition*, R A. R A + P A = R P : : R V. R V + V A = R A; wherefore R A. R P : : R V. R A; therefore R A*q* = R P × R V.

NOW, *as in the firſt Propoſition*, becauſe of the Circle M S D A, F D. F A : : F M. F S, and *their Halves* C D. R A : : R P. C O; but C D. R A : : C H. R V, and *ex æquo* R P C O : : C H: R V. and the Rectangles under the Extreams and Means being equal, C H × C O is = R P × R V; but the Rectangle R P × R V *was demonſtrated above* to be equal R A*q*, and R A = C T by the *Conſtruction*; wherefore

$CTq = CH \times CO$ therefore CH. CT :: CT. CO. Q. E. D.

COROLLARY.

SINCE CH. CT :: CT. CO by *inverting the Demonſtration of the firſt Article of this Prop.* it may be proved, that IO. OT :: IH. HT, and the Line IO is ſaid to be divided *harmonically* in the Points I, O, T, H.

A TREATISE OF CONICK SECTIONS.

PART IV.

The Deſcriptions of the CONICK SECTIONS *upon a PLANE.*

THE Method of deſcribing *Curve Lines* upon a Plane, by the continual Motion of a Point with Machines, is ſo ſubject to Error, that it ought not to be uſed above once, leſt its Perplexity ſhould diſcourage the Learner; and I am apt to think there is nothing more requiſite than to find an infinite Number of Points, after an eaſy manner, through which may be drawn the *Curve Line* deſired; and as it frequently happens that there is occaſion but for a ſmall Part of a *Curve Line*, ſo there may be found ſo great a Number of Points, that there can be no conſiderable Error in drawing the *Curve* through the Points ſo found. The Deſcriptions of the three *Conick Sections* which I have given at the Beginning of the Demonſtrations of every one, being generally received for the moſt ſimple, when the *Foci* or *Axes* are known, it would be needleſs to ſeek for any other, when thoſe Things are given. But becauſe it often happens, that one may have Occaſion from other *Date* to deſcribe the *Sections*; for Inſtance, to deſcribe the *Parabola*, having given only one of the *Diameters*, with its *Parameter*, and the Angle the *Ordinates* make with that *Diameter*. To deſcribe the *Ellipſis*, having given two Conjugate Diameters; and to deſcribe the *Hyperbola*, having given the Angle made by the *Aſſymptotes* and any Point in the *Curve*, or a *Diameter*, and its *Parameter*; or in one Word, to deſcribe any of the three *Sections*, having given a *Diameter* with one of its *Ordinates*. I have therefore given the Deſcriptions of theſe Lines in the Caſes propoſed.

PROB. I.

A Diameter of a Conick Section *being given with an* Ordinate *to that* Diameter *to find the* Parameter; *and in the* Ellipſis *to determine its* conjugate Diameter.

For the Parabola.

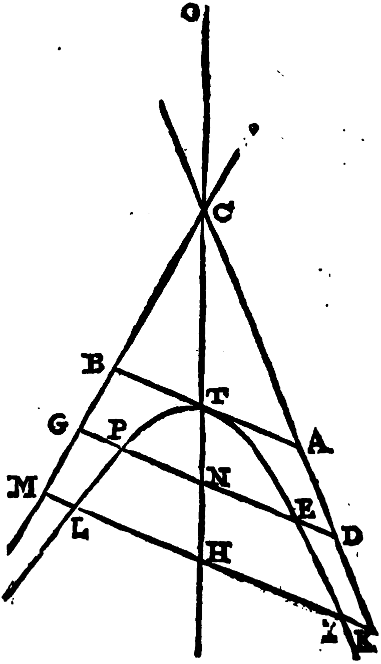
IF you ſay, as the intercepted Diameter is to the Ordinate, ſo is the Ordinate to a third Proportional. *It is evident* by the Definition of the *Parameter* of a Diameter in the *Parabola*, that this third Proportional is the *Parameter* ſought.

For the Ellipſis *and* Hyperbola.

IF you make as the Rectangle under the Parts of that Diameter, made by the Interſection of the Ordinate to this Diameter, is to the Square of that Ordinate, ſo is the Diameter to a fourth Proportional, which fourth Proportional thus found will be the *Parameter* of that Diameter: And in the *Ellipſis* having found a mean Proportional between the Diameter, and its *Parameter*. And having drawn thro' the Centre a Line parallel to the Ordinate, and equal to the mean Proportional found, and which is biſſected by the Centre, we ſhall have the conjugate Diameter to the Diameter propoſed; as is evident *by the Definition of the* Parameter *of a Diameter in the* Ellipſis *and* Hyperbola, *and by Prop.* xvii. *and* xviii. *of the* Ellipſis.

PROB. II.

In an Hyperbola, *a determinate Diameter with its* Parameter *being given, and the Angle which this Diameter makes with its* Ordinate, *to deſcribe the* Aſſymptotes.

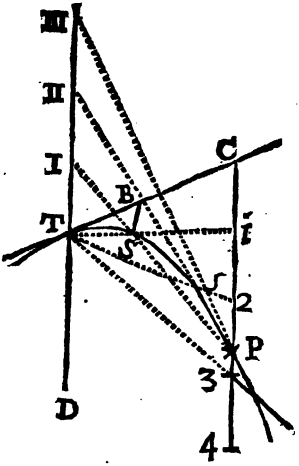


LET the Diameter given be O T, and the Angle O N E, which this Diameter makes with its Ordinate; through the Extremity T draw A T B parallel to E N; and having found A B, a mean Proportional between the Diameter O T and its *Parameter*, biſſects it in T; and through the Centre, and the Points A and B, produce C A and C B on each Side the Centre C. I ſay, that C A, C B are the *Aſſymptotes* of the propoſed *Hyperbola*; *as is evident by the Generation of the* Parameter *of an* Hyperbola.

PROB. III.

The Right Line I T D *being given for the Diameter of a* Parabola, *and the Point* P *for one of its Points, together with the Line* C T *touching it at* T, *the Extremity of that Diameter: Or rather the Angle* D T C = *to the Angle the Diameter makes with its Ordinate; to deſcribe the* Parabola.

THROUGH the Point P having drawn C P parallel to T D, and prolonged it downwards from P, take C, *i*, *i*, 2, 23, &*c.* equal to one another, of what Bigneſs you pleaſe, ſo you do but begin at the Point C where CP meets TC; produce TD upwards, and ſet off the ſame Parts from T,



making them thus, TI, I II, II, III, *&c.* Having drawn PI, T*i* meeting one another in S, I ſay, the Point S is one of the Points of the *Parabola* required.

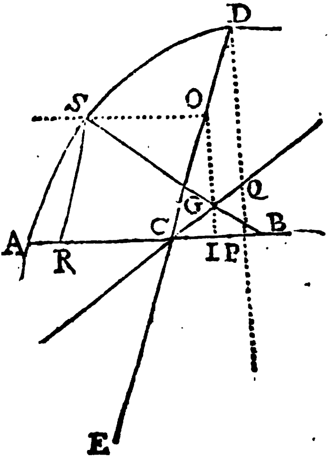
DRAW SB parallel to DT; and *from the ſimilar Triangles* TBS, TC*i*, and ITS, P*i* $SBS.\ Ci = TI :: TB.\ TC$; $TI.\ iP + TI$ or $Ci = CP :: TS.\ Ti :: TB.\ TC$ wherefore BS. CP in a duplicate Ratio of TB to TC; But likewiſe

T B*q* : T C*q* in a duplicate Ratio of their Sides T B, T C; wherefore T B*q*. T C*q* :: B S. C P; and the Point P being one of the Points of the *Parabola*, it is evident, by *Cor.* iii. *Prop.* i. *of the* Parabola, that S will alſo be a Point in the *Parabola* required.

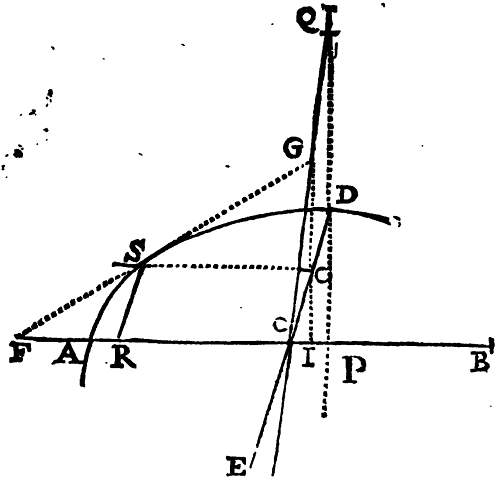
In the ſame Manner it may be demonſtrated that S the Interſection of P II and T 2, and Z that of P III and T 3, and ſo on, will be Points in the ſame *Parabola*; and if the Parts C *i*, &*c*. be taken very ſmall, you may find three Points very near one another.

PROP. IV.

The Right Lines A B, E D, *being given for the conjugate Diameters of an* Ellipſis *interſecting one another in* C; *to deſcribe that* Ellipſis.



Through D the Extremity of any one of the Diameters, draw D P perpendicular to the other conjugate Diameter A B, and produce it both Ways; and upon this Line take D Q on either Side of D = C A =



$\frac{1}{2}$ AB; through the Centre C, and the Point Q, draw the Line CQ produced both ways from C; from any Point O of the Diameter DE having drawn the right Lines OS parallel to CA, and OG parallel to DQ; and upon the Centre G, and with the Radius GS = CA having deſcribed the Ark S meeting OS in S: I ſay, the Point S, is one of the Points of the *Ellipſis* required.

FROM *the ſimilar Triangles* CDQ. COG. CDq. COq = RSq :: DQq = GSq = CAq. OGq. But

OGq = Difference of GSq and SOq; for the Triangle GOS is Rectangled in O, and *after the ſame Manner*, GOq is = Difference of the Squares of CA and CR, which are equal to GS and SO; and the Difference of CAq and CRq is = Rectangle BRA; for AB is biſſected in C, wherefore CDq. RSq :: CAq. BRA and (*by Prop.* xviii.) the Point S is a Point of the *Ellipſis* required, and ſo may an infinite Number of other Points be found.

Another Manner of Deſcription for the ſame Conſtruction.

HAVING drawn CQ as above, and taking G, ſome Point in the Line CQ, for a Centre, and with the Radius GF = QP, having deſcribed the Ark F cutting CB in F, and having prolonged FG to S, and drawn GS = CA; I ſay, the Point S is a Point in the *Ellipſis* required.

THROUGH the Point G having drawn IGO parallel to DQP; FG = PQ. GI :: DQ = GS. GO; wherefore if SO be drawn, it will be parallel to CA; *and it may be demonſtrated as above*, that the Point S is a Point in the *Ellipſis* ſought.

If the Lines AB, DE *be the* Axes.

IF the Lines AB, DE be the *Axes*, the preceding Method wlll ſerve; for *it is evident* that the Lines CQ and CD will be joined, and that QP will be the Difference *in the 1ſt Fig.* and the Sum *in the 2d Fig. in the preceding Prob.* of the two Semi-axes; and the Points, as G, ought to be taken upon CD; and *the reſt of the Conſtruction and Demonſtration will be the ſame as before.*

Another Manner of Deſcription by the Help of a Circle.

IN *the preceding Figure, the Reader muſt ſupply the Circle, and the Lines which are not there; which is very eaſy to do.*

If the Quadrant of a Circle be deſcrib'd with the Radius CA, and having joined AD; if from any Point R of the Diameter CA you erect a Perpendicular, and continue it to the Circumference of the Circle, this Perpendicular will be a mean Proportional between BR and RA; and having drawn RO parallel to AD, and OS parallel to CA, = to the mean Proportional between BR, RA: I ſay the Point S is in the *Ellipſis* ſought.

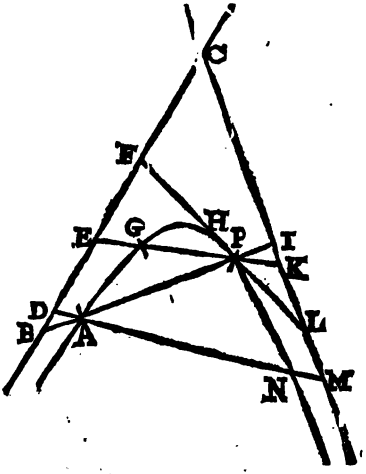
By the Conſtructions CD*q*. CA*q*:: Rectangle EOD. Rectangle BRA. OS*q*; wherefore CD*q*. CAQ or ED*q*. BA*q*:: Rectangle EOD. OS*q*; and (*by* 18 *Prop. Ellip.*) the Point S is a Point in the *Ellipſis* ſought.

Note, *in this Suppoſition* OSCR *are not* = *as in the Figure.*

PROB. V.

The Aſſymptotes CD, CM *being given with a Point in the* Hyperbola; *it is required to deſcribe the* Hyperbola.

THROUGH the Point P, draw BPI, EPK, FPL at pleaſure, terminated at the *Aſſymptotes*; and make BA = PI, EG = PK. FH = PL, I ſay, the Points A, G, H, are in the *Hyperbola* required; this is evident *by the* 13. *of the* Hyperbola.



AND if through any of the Points found you ſtill draw other Lines as DM, make MN = DA, and it is evident *for the ſame Reaſon*, that the Point N is in the *Hyperbola*.

FINIS.

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